Data reconstruction of rotating turbulent flows with Gappy POD and Generative Adversarial Networks

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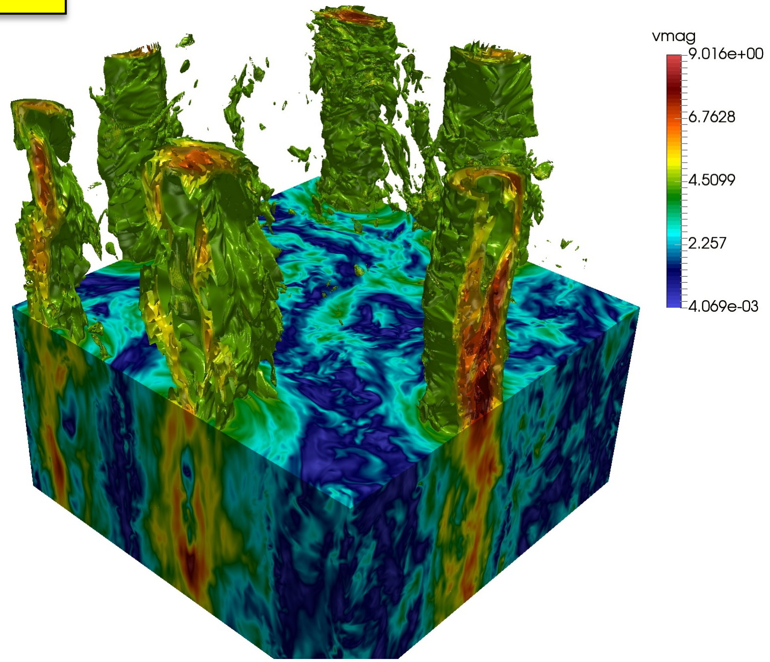
$$\begin{cases} \partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} + \nabla p - \nu\nabla^2\boldsymbol{u} = 2\boldsymbol{u}\times\boldsymbol{\Omega} + \boldsymbol{f} \\ \boldsymbol{\nabla}\cdot\boldsymbol{u} = 0 \end{cases}$$

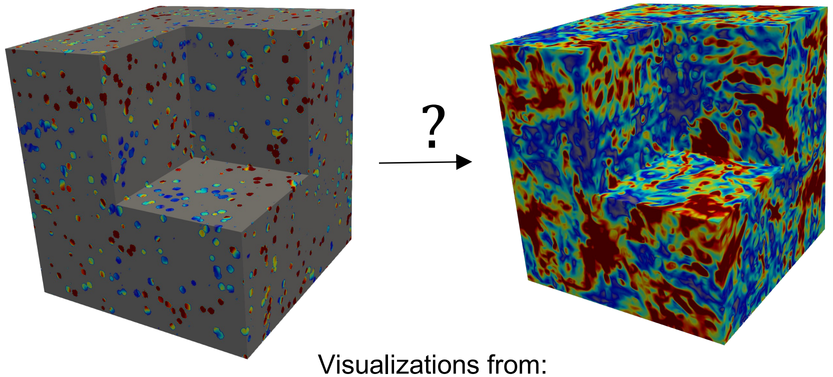
Turbulence on a rotating frame

Large quantity of high quality data



Data-Assimilation/Reconstruction

Reconstruction from partial measures



Visualizations from:

Clark Di Leoni, P., Mazzino, A., & Biferale, L. (2020). Physical Review X, 10(1), 011023.

T. Li, **Michele Buzzicotti**, F. Bonaccorso, Luca Biferale, (University of Rome Tor Vergata), S. Chen, M. Wan (Shenzhen University)

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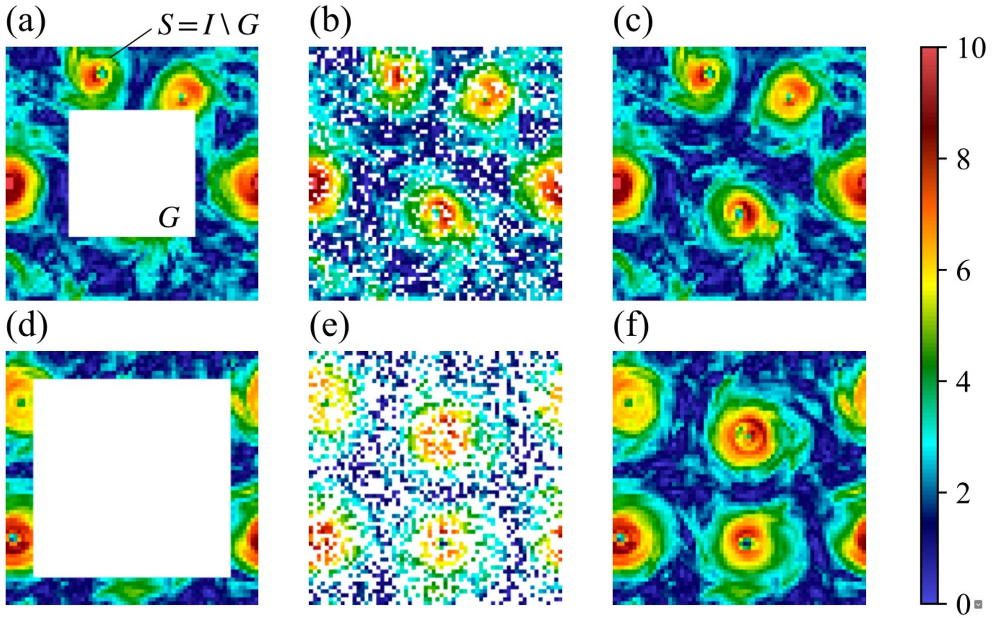
Full State RECONSTRUCTION

Problem Setup:

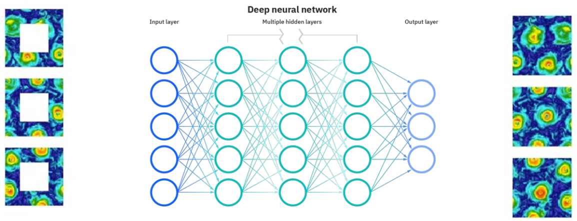


DATA DRIVEN

$$u(\boldsymbol{x}) = \sum_{n=1}^{N} a_n \psi_n(\boldsymbol{x}) = \sum_{n=1}^{N'} a_n \psi_n(\boldsymbol{x}) + \sum_{n=N'+1}^{N} a_n \psi_n(\boldsymbol{x})$$

$$\int_S \left[u(\boldsymbol{x}) - \sum_{n=1}^{N'} a_n \psi_n(\boldsymbol{x}) \right]^2 \mathrm{d}\boldsymbol{x}$$

1. EQUATION FREE
PRINCIPAL ORTHOGONAL DECOMPOSITION
GAPPY-POD & EXTENDED POD



2. EQUATION FREE
GENERATIVE-ADVERSARIAL-NETWORK

EQ. BASED

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} \boldsymbol{v} + 2\Omega \hat{x}_3 \times \boldsymbol{v} = -\boldsymbol{\nabla} p + \nu \nabla^2 \boldsymbol{v} + \gamma (1 - \hat{M})_{x_3} \odot (\boldsymbol{v} - \boldsymbol{v}_{\mathrm{ref}})$$

3. NUDGING

GAPPY-POD (PRINCIPAL ORTHOGONAL DECOMPOSITION)

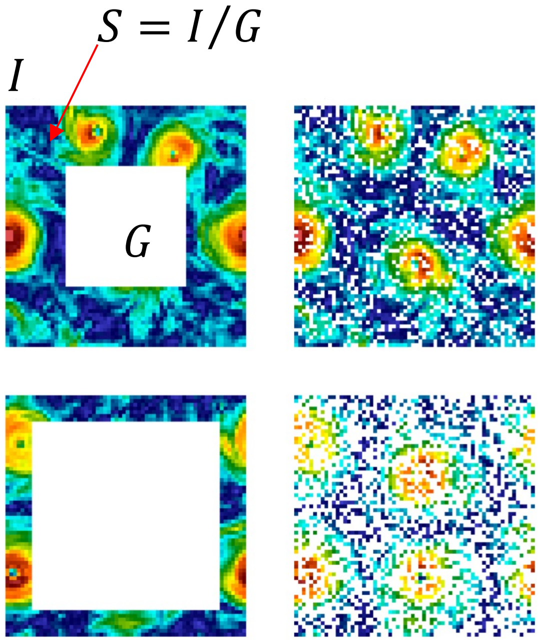
DATA DRIVEN

$$u_{pred}(\mathbf{x}) = \sum_{n=1}^{N} a_n\psi_n(\boldsymbol{x}) = \sum_{n=1}^{N'} a_n\psi_n(\boldsymbol{x}) + \sum_{n=N'+1}^{N} a_n\psi_n(\boldsymbol{x}), \quad (\boldsymbol{x} \in G)$$

$$K(\mathbf{x},\mathbf{y}) = \langle u_{true}(\mathbf{x})u_{true}(\mathbf{y})\rangle \qquad (\boldsymbol{x},\boldsymbol{y} \in I)$$

$$\int K(\mathbf{x},\mathbf{y})\psi_n(\mathbf{y})d\mathbf{y} = \lambda_n\psi_n(\mathbf{x})$$

$$\tilde{E} = \int_S d\boldsymbol{x}\left(u(\boldsymbol{x}) - \sum_{n=1}^{N'} a_n\psi_n(\boldsymbol{x})\right)^2$$



LINEAR OPTIMAL REGERESSION

Karhunen–Loeve procedure for gappy data.
R. Everson & L. Sirovich,
JOSA A, *12*(8), 1657-1664, 1995

Data reconstruction of turbulent flows with Gappy-POD and Generative Adversarial Networks
T. Li, M. Buzzicotti, F. Bonaccorso, L.B., S. Chen, M. Wan .
arXiv preprint arXiv:2210.11921 (Submitted Journal Fluid Mechanics), 2022

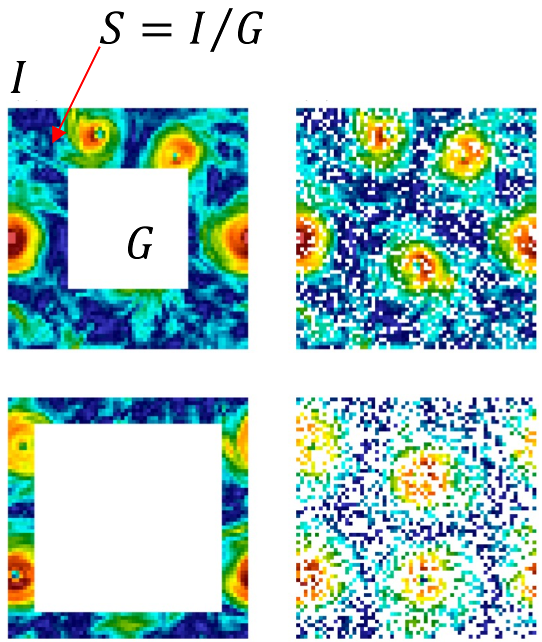
EXTENDED POD (PRINCIPAL ORTHOGONAL DECOMPOSITION)

DATA DRIVEN

$$u_{pred}(\boldsymbol{x}) = \sum_{n=0}^{\tilde{N}} b_n \phi_n^e(\boldsymbol{x}) = \sum_{n=0}^{\tilde{N}} \overbrace{\int_S \left(\phi_n(\boldsymbol{x}') \cdot u(\boldsymbol{x}')\right) d\boldsymbol{x}'}^{b_n} \phi_n^e(\boldsymbol{x}) \quad (\boldsymbol{x} \in G)$$

$$K(\mathbf{x}, \mathbf{y}) = \langle u_{true}(\mathbf{x}) u_{true}(\mathbf{y}) \rangle \qquad (\boldsymbol{x}, \boldsymbol{y} \in S)$$

$$\int K(\mathbf{x}, \mathbf{y})\, \phi_n(\mathbf{y}) d\mathbf{y} = \lambda_n\, \phi_n(\mathbf{x}) \; (\boldsymbol{x}, \boldsymbol{y} \in S)$$



POD MODES

$$\phi_n(\boldsymbol{x}) = \frac{\langle b_n u(\boldsymbol{x}) \rangle}{\lambda_n} \qquad (\boldsymbol{x} \in S)$$

EPOD MODES

$$\phi_n^e(\boldsymbol{x}) = \frac{\langle b_n u(\boldsymbol{x}) \rangle}{\lambda_n} \qquad (\boldsymbol{x} \in G)$$

Extended proper orthogonal decomposition: a tool to analyse correlated events in turbulent flows.
Borée, J.
Experiments in fluids, 35(2), 188-192.

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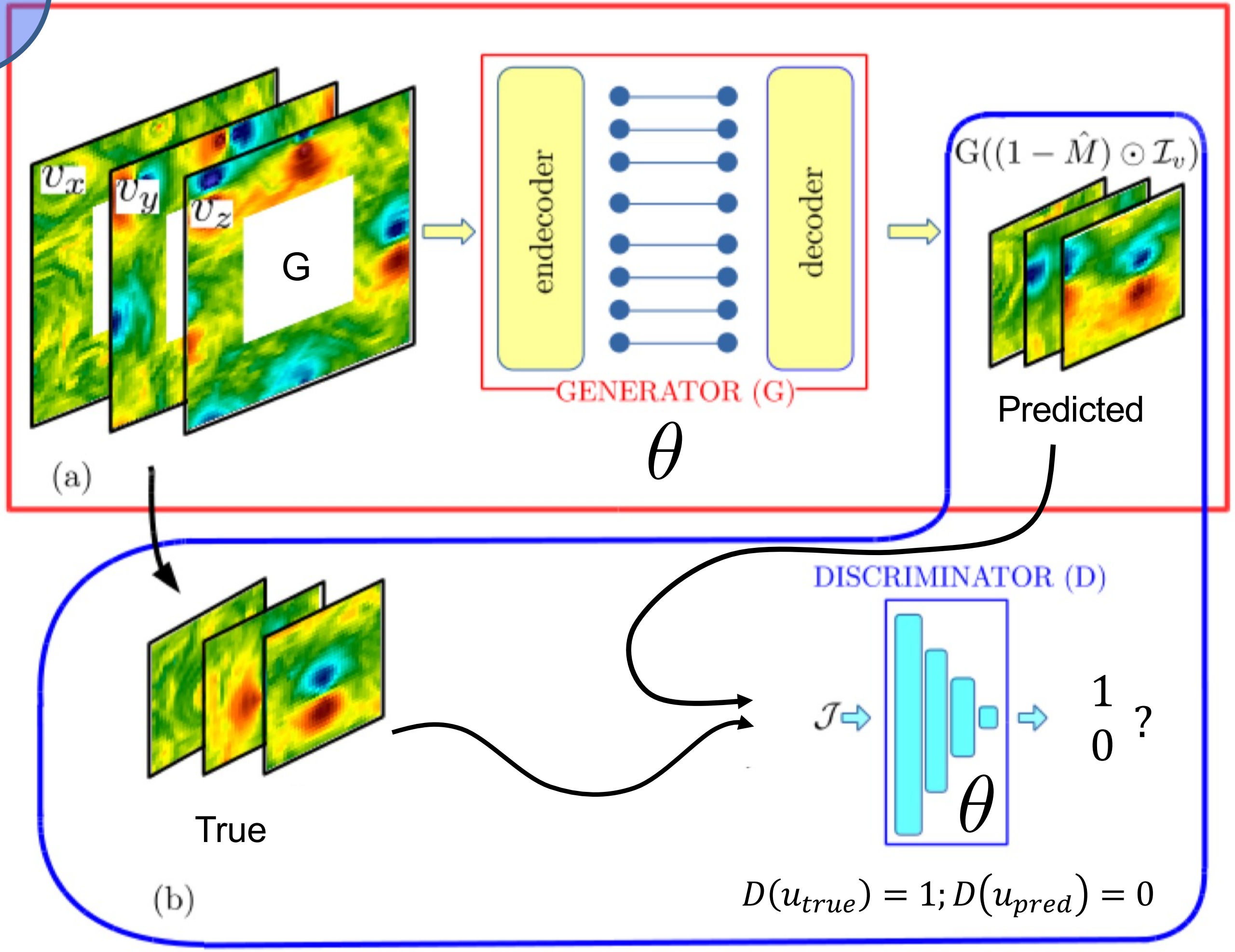
GENERATIVE ADVERSARIAL NETWORK:

DATA DRIVEN



Reconstruction of turbulent data with deep generative models for semantic inpainting from TURB-Rot database
M. Buzzicotti, F. Bonaccorso, P. Clark Di Leoni, and L. B.
Phys. Rev. Fluids **6**, 050503 , May 2021

MINIMIZE:

$$\mathcal{L}_G = \langle \int_G d\mathbf{x}(u_{true}(\mathbf{x}) - u_{pred}(\mathbf{x},\theta))^2 \rangle$$

$$\mathcal{L}_{adv} = \log\left(1 - D\left(u_{\text{pred}}\right)\right).$$

$$\mathcal{L}_{TOT} = \mathcal{L}_G + \lambda\, \mathcal{L}_{adv}$$

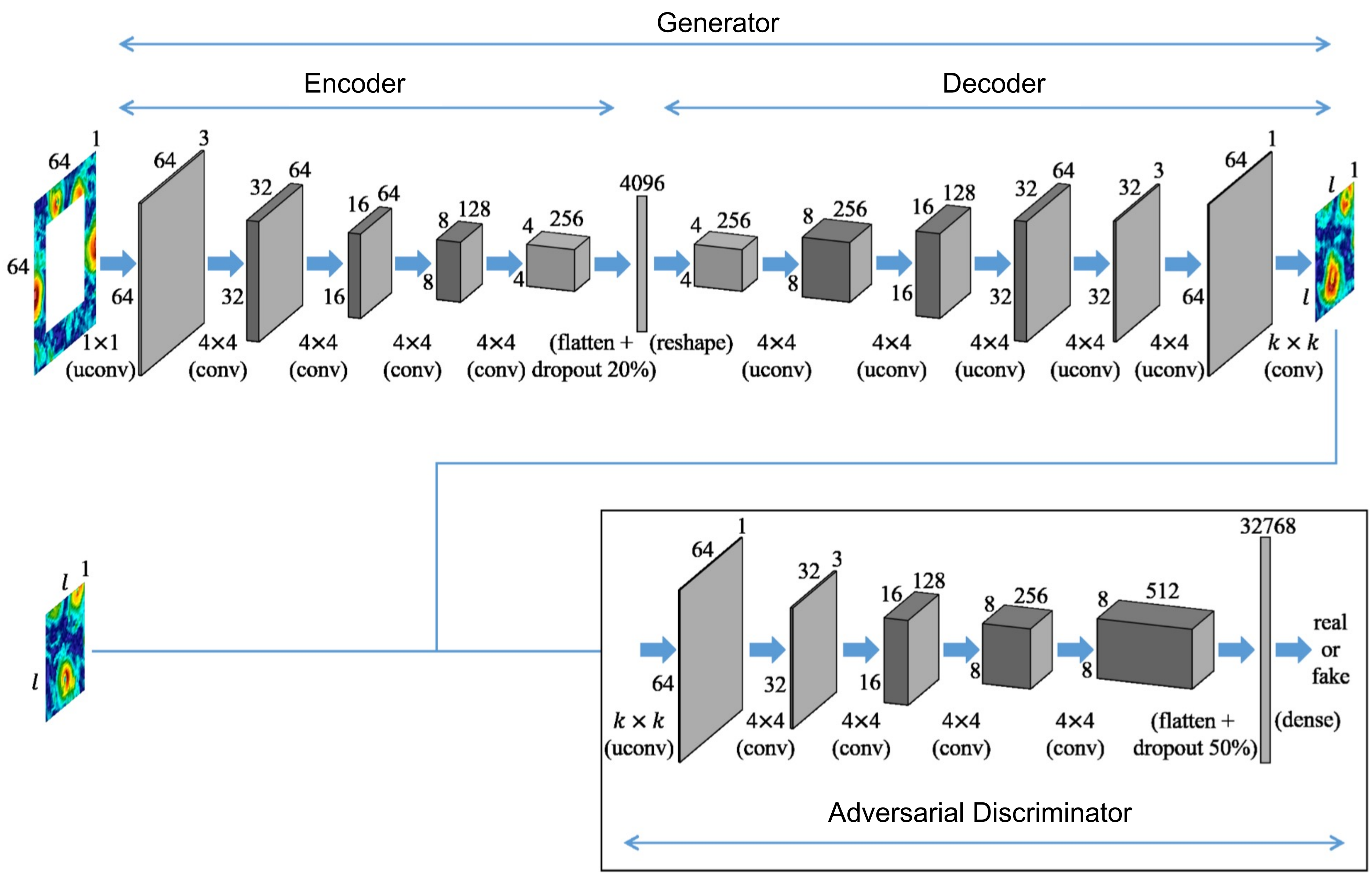
MAXIMIZE:

$$\mathcal{L}_{DIS} = \log\left(D\left(u_{\text{true}}\right)\right) + \log\left(1 - D\left(u_{\text{pred}}\right)\right).$$

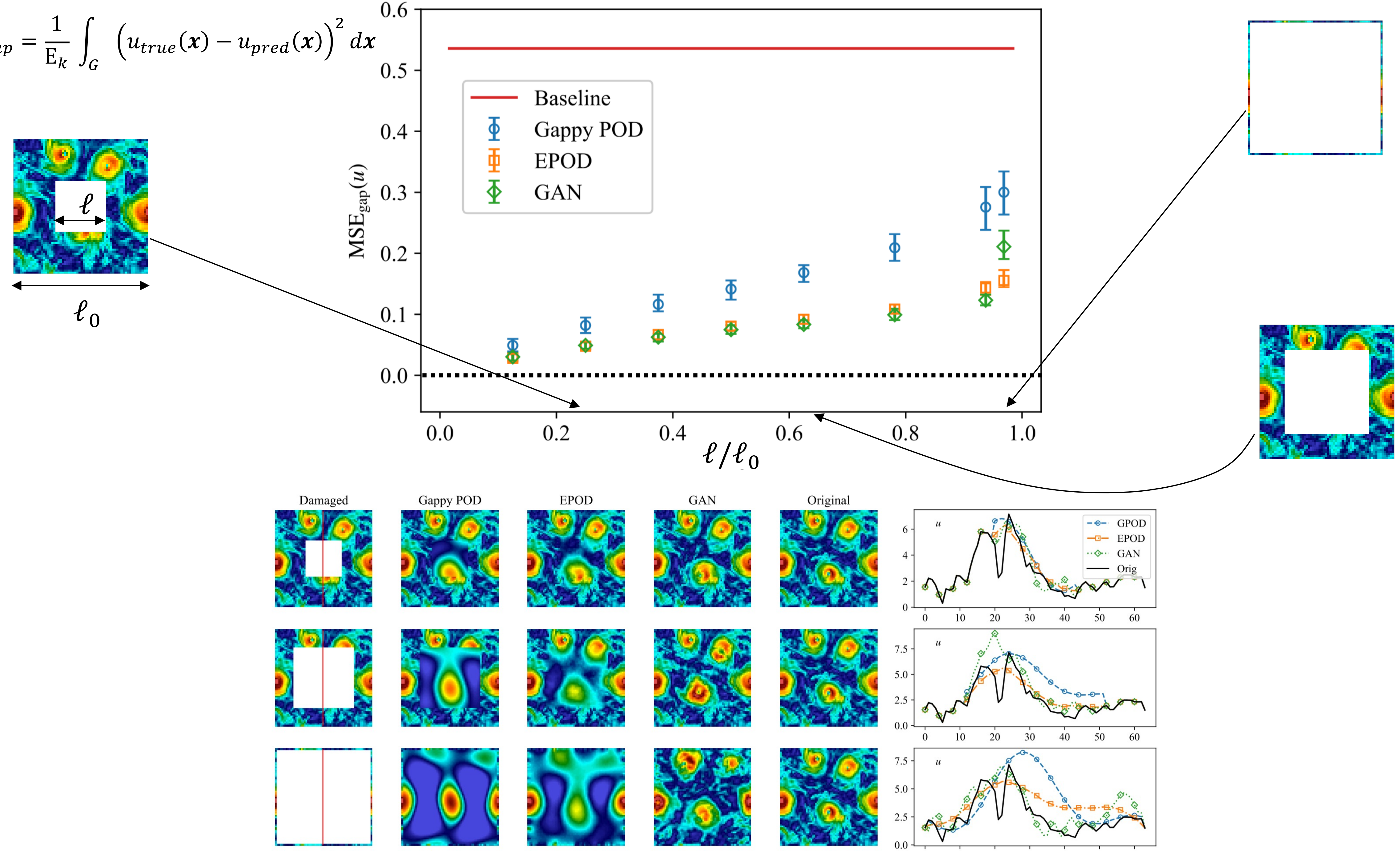
Context encoders: Feature learning by inpainting
D. Pathak, P. Krahenbuhl, J. Donahue, T. Darrell, and A. Efros.
Proceedings of the IEEE conference on computer vision and pattern recognition. 2016.

DATA DRIVEN

GENERATIVE ADVERSARIAL NETWORK



$$MSE_{gap} = \frac{1}{E_k} \int_G \left(u_{true}(\boldsymbol{x}) - u_{pred}(\boldsymbol{x})\right)^2 d\boldsymbol{x}$$



Comparison of Statistical Fluctuations

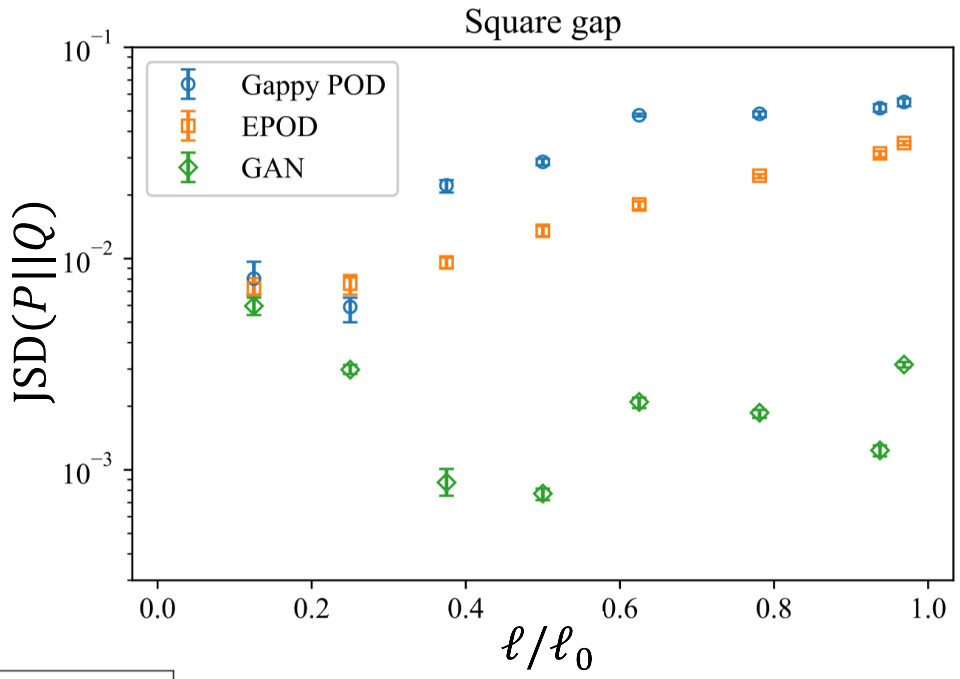
$$D(P \parallel Q) = \int_{-\infty}^{\infty} P(x) \log\left(\frac{P(x)}{Q(x)}\right) \mathrm{d}x$$

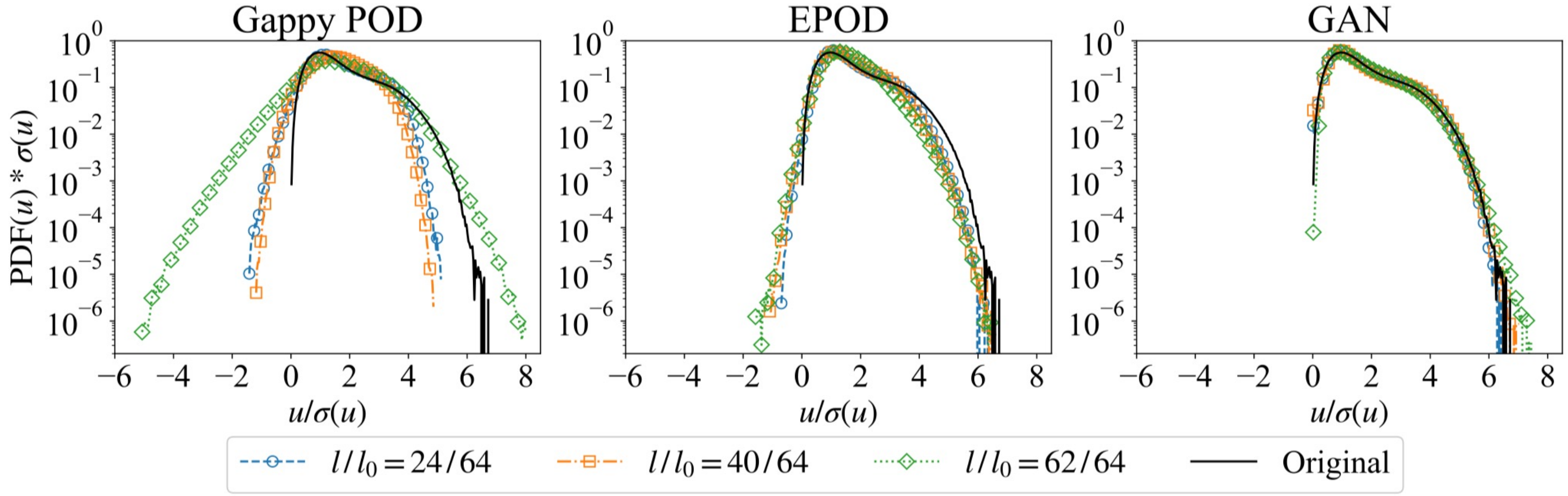
$$M = \frac{1}{2}(P + Q)$$

KULLBACK-LEIBLER Divergence

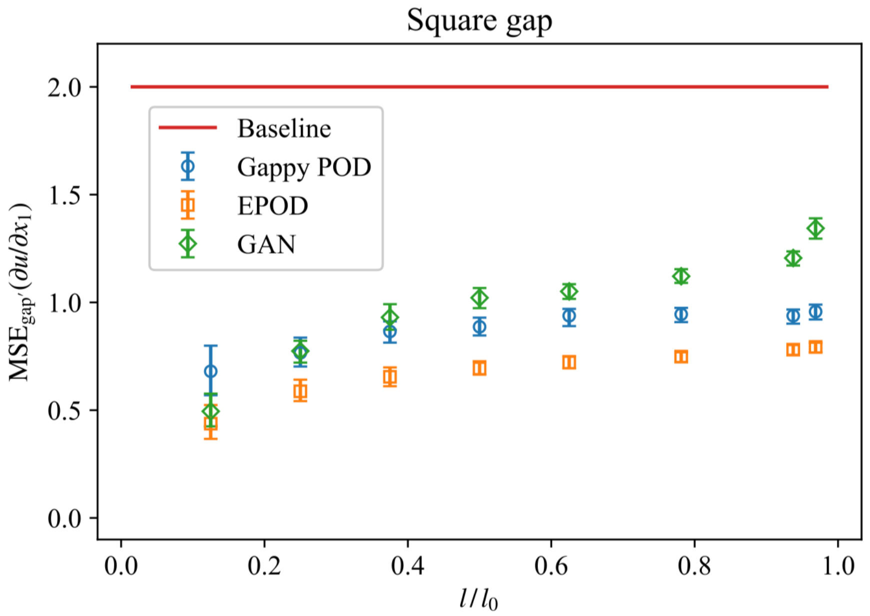
JENSEN-SHANNON Divergence

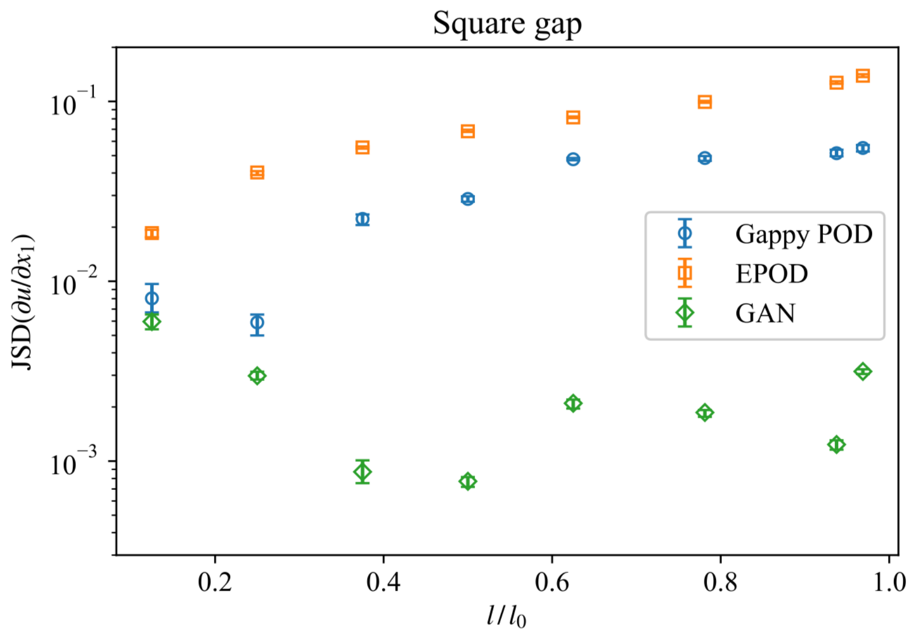
$$\mathrm{JSD}(P \parallel Q) = \frac{1}{2} D(P \parallel M) + \frac{1}{2} D(Q \parallel M),$$

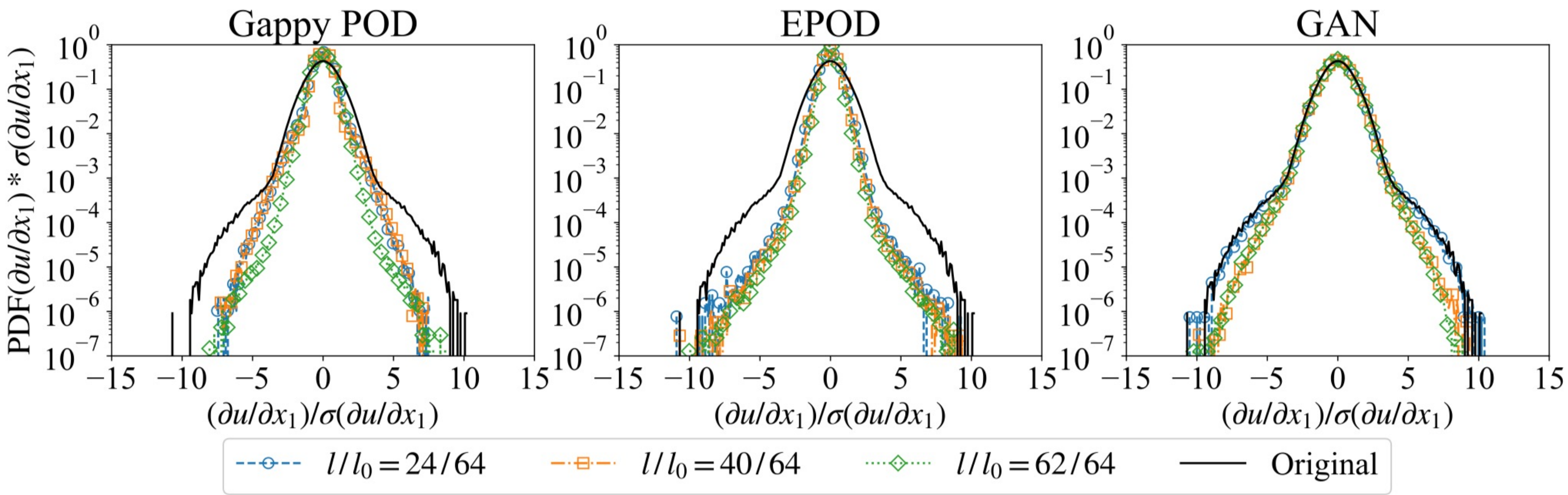




Comparison of Statistical Fluctuations at small scales (velocity gradients)

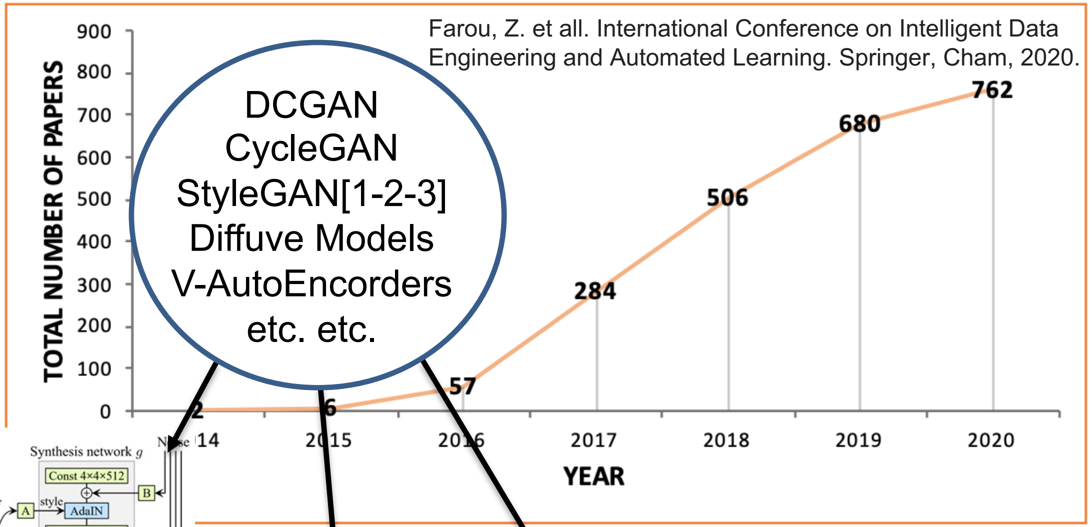






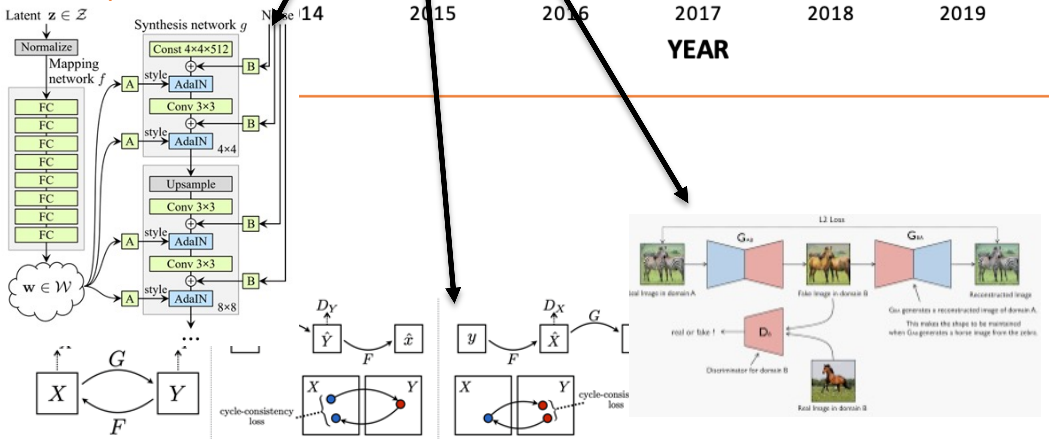
Two fundamental Open Questions:

(i) How can we find/fine-tune the best data-driven strategy or architecture?



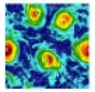
Farou, Z. et all. International Conference on Intelligent Data Engineering and Automated Learning. Springer, Cham, 2020.

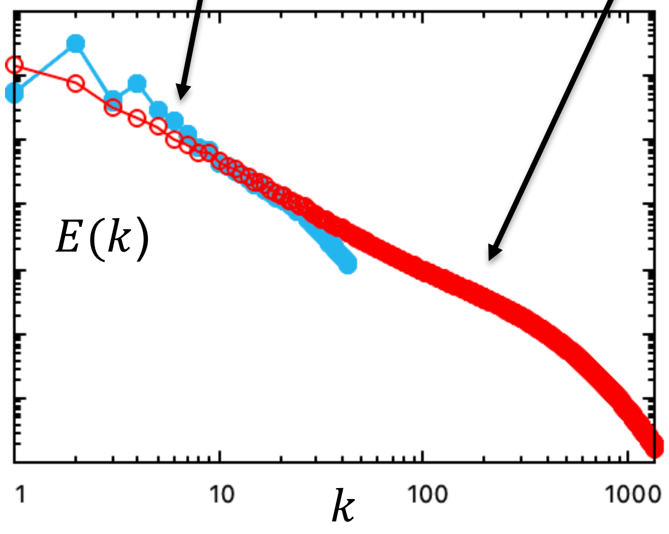
DCGAN
CycleGAN
StyleGAN[1-2-3]
Diffuve Models
V-AutoEncorders
etc. etc.

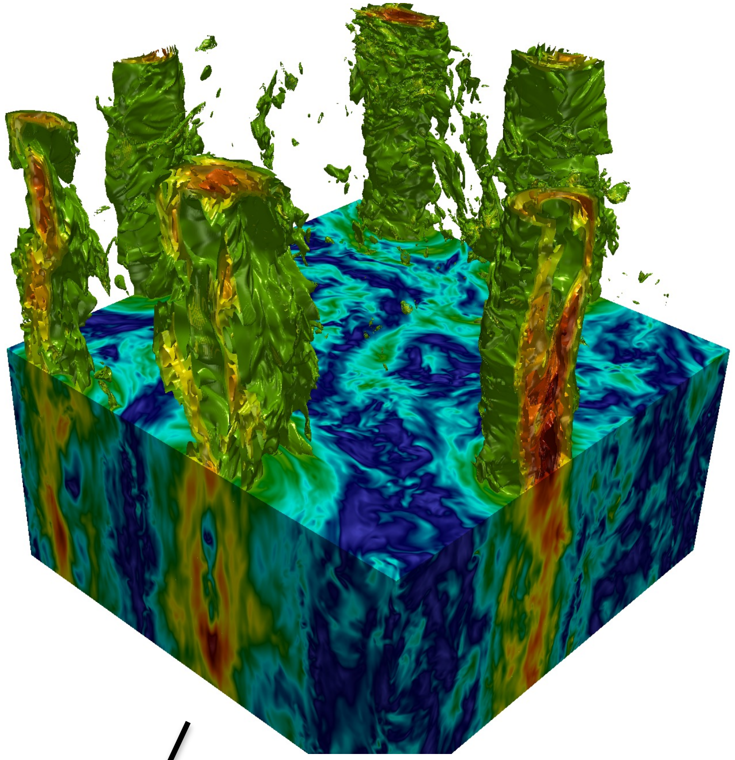


(ii) Can we push the analysis on state-of-the-art datasets?

128×128

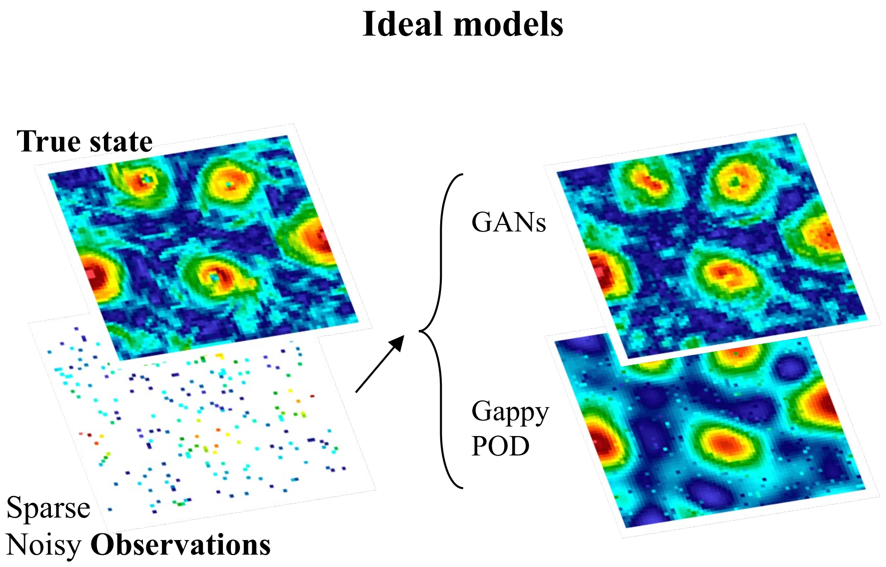






4096× 4096× 4096

CONTRIBUTIONS AND FUTURE PERSPECTIVES



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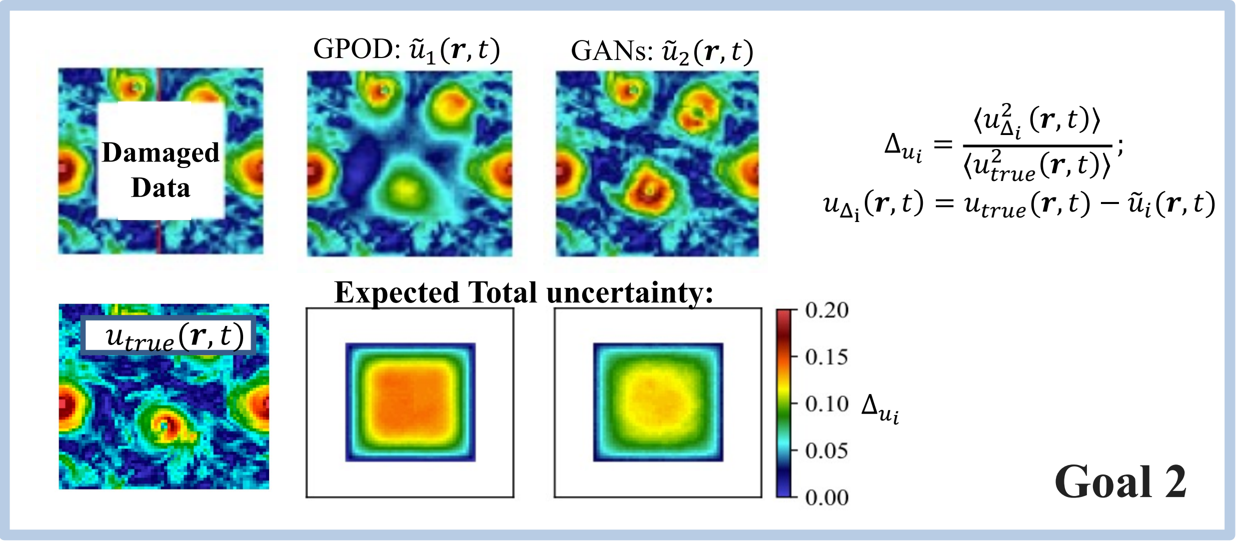
arXiv:2210.11921 (Submitted Journal Fluid Mechanics), 2022

Uncertainty Quantification



Application to Observation-data