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Phase transition to large scale coherent structures in 2d active matter turbulence

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Phase transition to large-scale coherent structures in 2D active matter turbulence

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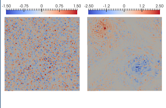

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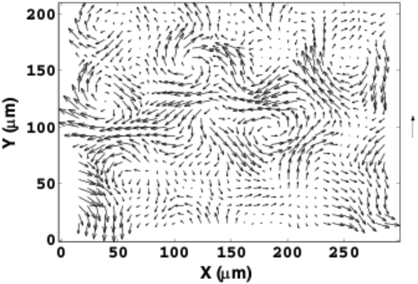
01.06.2018

Motivation: bacterial turbulence



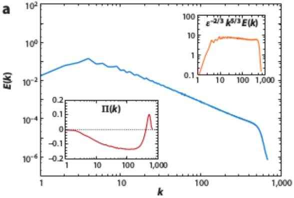
Velocity field of pendant drop of *Bacillus subtilis* suspension
Dombrowski et al,
PRL **93** 098103 (2004)

Collective effects of many microswimmers lead to vortices larger than the characteristic size of an individual microswimmer

- length of microswimmer, e.g. *B. subtilis*: $O(5\mu m)$
- size of vortex: $O(100\mu m)$

From bacterial turbulence to hydrodynamic turbulence

- (quasi)-2D bacterial suspensions: bacterial films, plankton
- 2D turbulence: inverse energy cascade



Energy spectrum and flux in 2D turbulence
Boffetta, ARFM (2014)

- local Re for microswimmer $O(10^{-5})$: dissipative range
- pusher type bacteria can lower effective viscosity

Hatwalne et al., PRL **92** 118101 (2004), Liverpool & Marchetti PRL **97** 268101 (2006), Sokolov & Aronson, PRL **103** 148101 (2009), López et al, PRL **115** 028301 (2015)

1. transition to hydrodynamic turbulence?
2. sustained turbulent cascade for strong driving?

Outline

Models

1. Bacterial flow model

   Wensink et al. PNAS **109** 14308 (2012)

   Bratanov et al. PNAS **53** 15048 (2015)

$$\partial_t \boldsymbol{u} + \lambda_0 (\boldsymbol{u} \cdot \nabla) \boldsymbol{u} + \nabla p = \lambda_1 \nabla |\boldsymbol{u}|^2 - (\alpha + \beta |\boldsymbol{u}|^2) \boldsymbol{u} + \Gamma_0 \Delta \boldsymbol{u} - \Gamma_2 \Delta^2 \boldsymbol{u},$$

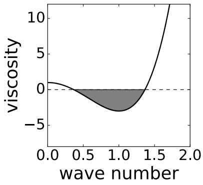
2. Solvent model

   Słomka & Dunkel EPJE **224** 1349 (2015)

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla) \boldsymbol{u} + \nabla p = \nabla \cdot \sigma,$$

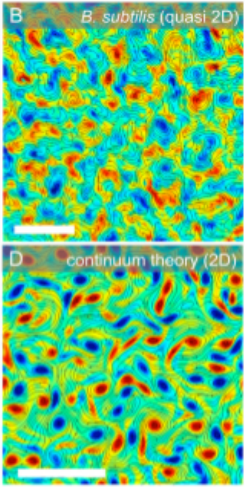
$$\sigma_{ij} = \left(\Gamma_0 - \Gamma_2 \Delta + \Gamma_4 \Delta^2 \right) (\partial_i u_j + \partial_j u_i)$$

$$\hat{\nu}_{\text{eff}} = \Gamma_0 + \Gamma_2 k^2 + \Gamma_4 k^4 \, .$$



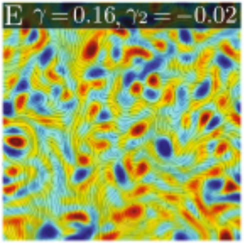
Comparison to experiments: Vorticity field

Wensink et al. (2012)



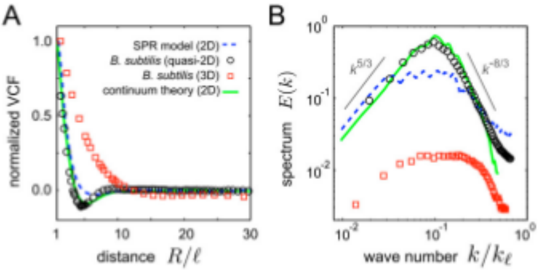
scale bar: $50\mu m$, domain length: $217\mu m$

Słomka & Dunkel (2015)



domain length: $600\mu m$

Comparison to experiments: energy spectrum



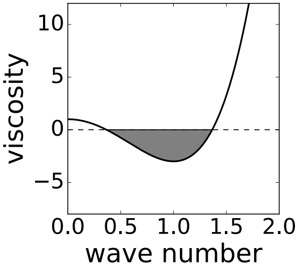
Wensink et al. PNAS **109** 14308 (2012)

Piecewise constant viscosity (PCV) model

$$\partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p + D[\boldsymbol{u}],$$

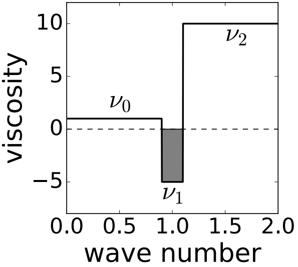
$$D[\boldsymbol{u}] = -\int_{\mathbb{R}^2} d\boldsymbol{k}\ \hat{\nu}(|\boldsymbol{k}|)k^2\hat{\boldsymbol{u}}_{\boldsymbol{k}}e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\ ,$$

continuous



Wensink et al. PNAS **109** 14308 (2012)
Bratanov et al. PNAS **53** 15048 (2015),
Słomka & Dunkel EPJE **224** 1349 (2015)

piecewise constant

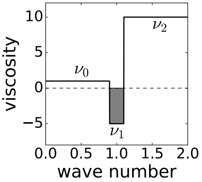


Piecewise constant viscosity (PCV) model

$$\partial_t \boldsymbol{u}(\boldsymbol{x}) + (\boldsymbol{u}(\boldsymbol{x}) \cdot \nabla)\boldsymbol{u}(\boldsymbol{x}) + \nabla p(\boldsymbol{x}) = \int_{\mathbb{R}^2} d\boldsymbol{y}\ \nu(\boldsymbol{x}-\boldsymbol{y})\Delta_{\boldsymbol{y}}\boldsymbol{u}(\boldsymbol{y})\ ,$$

$$\partial_t \hat{\boldsymbol{u}}(\boldsymbol{k}) + \int_{\mathbb{R}^2} d\boldsymbol{p}\ (\hat{\boldsymbol{u}}(\boldsymbol{p}) \cdot i\boldsymbol{k})\hat{\boldsymbol{u}}(\boldsymbol{k}-\boldsymbol{p}) + i\boldsymbol{k}\hat{p}(\boldsymbol{k}) = -\hat{\nu}(k)k^2\hat{\boldsymbol{u}}(\boldsymbol{k})\ ,$$

$$\hat{\nu}(k) = \begin{cases} \nu_0 > 0 & \text{for} \quad k < k_{\mathrm{min}}\ , \\ \nu_1 < 0 & \text{for} \quad k \in [k_{\mathrm{min}}, k_{\mathrm{max}}]\ , \\ \nu_2 > 0 & \text{for} \quad k > k_{\mathrm{max}}\ . \end{cases}$$



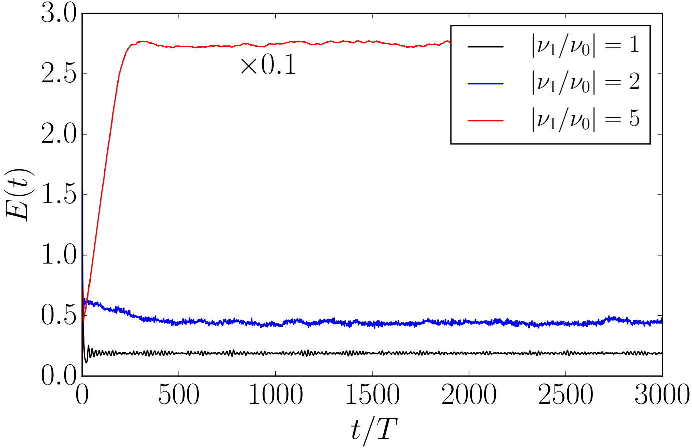
- Galilean invariant
- One single parameter, ν_1, controls the energy input.

Parameter study: numerical details

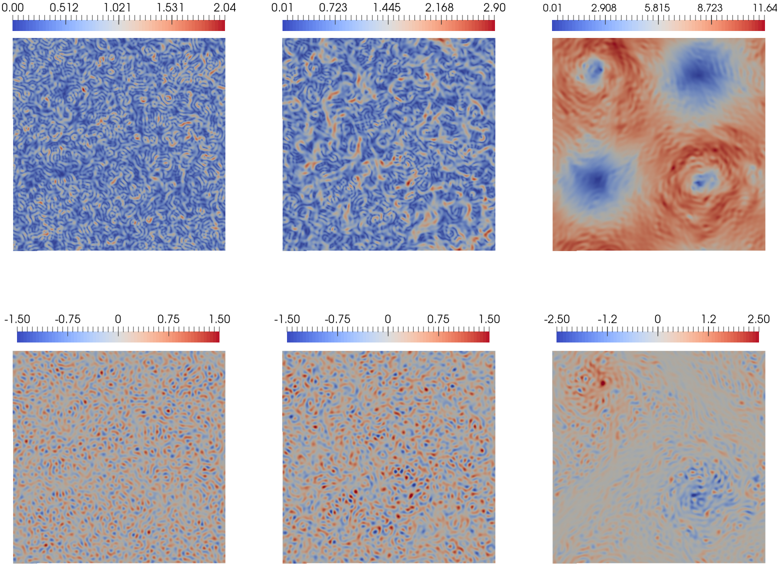
- fully dealiased pseudospectral DNS
- domain $[0, 2\pi]^2$, discretised on 256^2 and 1024^2 grid points
- keep ν_0 and ν_2 fixed
- keep amplification wavenumber band $[k_{\min}, k_{\max}]$ fixed
- vary amplification factor ν_1

$\nu_0 = 0.00109$
$\nu_2/\nu_0 = 10$
$k_{\min} = 33$
$k_{\max} = 42$
$|\nu_1/\nu_0| \in [0.25, 7]$

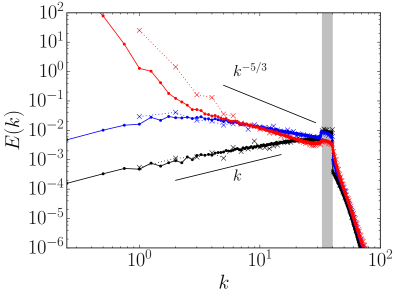
Parameter study: Time evolution

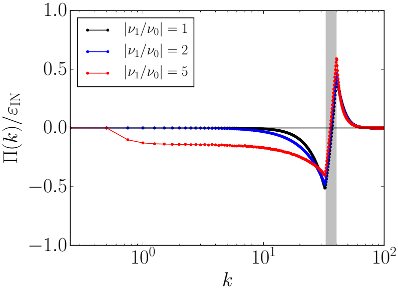


$|\nu_1/\nu_0| = 1$ $|\nu_1/\nu_0| = 2$ $|\nu_1/\nu_0| = 5$

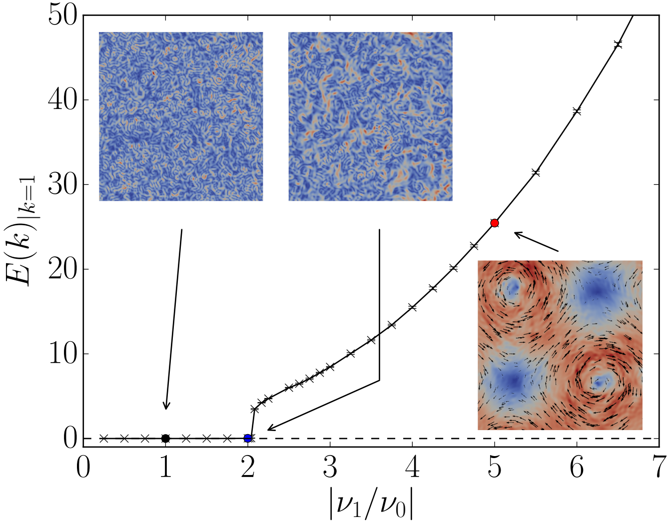


Parameter study: Scale-by-scale results

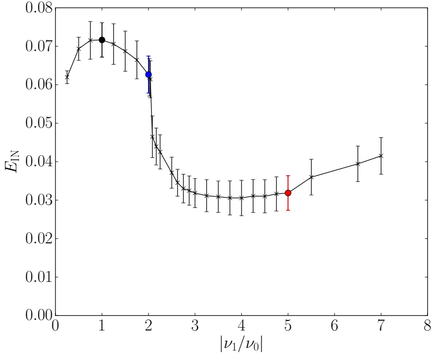


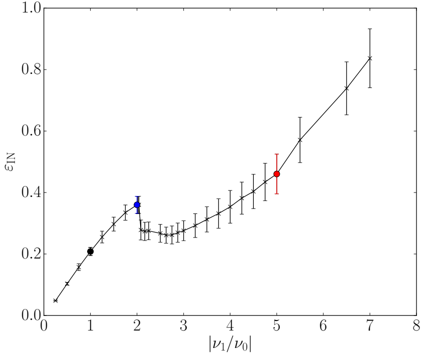


Parameter study: Global results



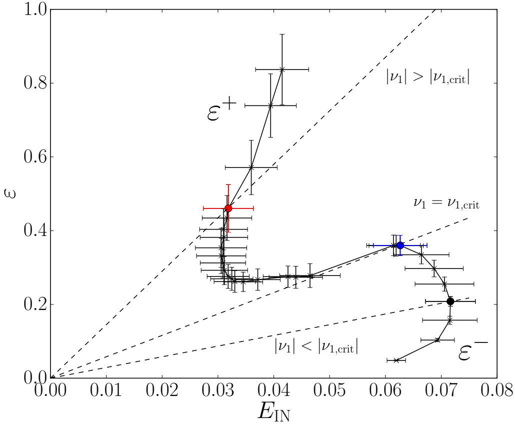
Parameter study: Global results



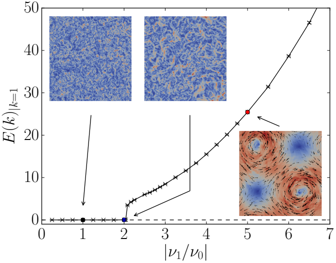


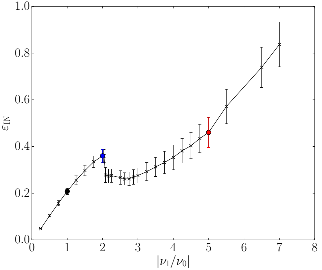
Parameter study: Global results

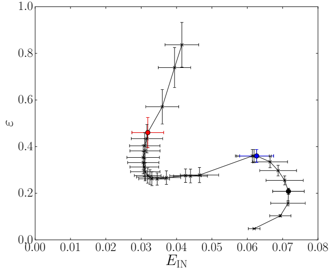
$$\varepsilon \simeq 2\nu_1 k_{\rm IN}^2 E_{\rm IN}$$



Parameter study: Global results







Four-scale model

- condensate: $E_1 := E(k)_{|k=1}$
- large scales: $E_{\mathrm{LS}} := \int_1^{k_{\min}} E(k)\, dk$
- driven scales: $E_{\mathrm{IN}} := \int_{k_{\min}}^{k_{\max}} E(k)\, dk$
- small scales: $E_{\mathrm{SS}} := \int_{k_{\min}}^{\infty} E(k)\, dk$

Interscale energy transfers

$$
\begin{aligned}
E_{\mathrm{IN}} &\longrightarrow E_{\mathrm{LS}} : \quad c_1 E_{\mathrm{LS}}^{1/2} E_{\mathrm{IN}} \ , \\
E_{\mathrm{IN}} &\longrightarrow E_1 : \quad c_2 \theta(E_1 - E_{1,0}) E_1^{1/2} E_{\mathrm{IN}} \ , \\
E_{\mathrm{LS}} &\longrightarrow E_1 : \quad c_3 E_1^{1/2} E_{\mathrm{LS}} \ , \\
E_{\mathrm{IN}} &\longrightarrow E_{\mathrm{SS}} : \quad c_4 E_{\mathrm{SS}}^{1/2} E_{\mathrm{IN}} \ ,
\end{aligned}
$$

$c_i > 0$: coupling constants determined from DNS

θ: Heaviside step function

Four-scale model

Evolution equations

$$\begin{aligned}\dot{E}_{\mathrm{IN}} &= -2\nu_1 k_{\mathrm{IN}}^2 E_{\mathrm{IN}} - c_1 E_{\mathrm{LS}}^{1/2} E_{\mathrm{IN}} - c_4 E_{\mathrm{SS}}^{1/2} E_{\mathrm{IN}} \\ &\quad - c_2 \theta(E_1 - E_{1,0}) E_1^{1/2} E_{\mathrm{IN}}\end{aligned}$$

$$\dot{E}_{\mathrm{LS}} = -2\nu_0 k_{\mathrm{LS}}^2 E_{\mathrm{LS}} + c_1 E_{\mathrm{LS}}^{1/2} E_{\mathrm{IN}} - c_3 E_1^{1/2} E_{\mathrm{LS}}$$

$$\dot{E}_{\mathrm{SS}} = -2\nu_2 k_{\mathrm{SS}}^2 E_{\mathrm{SS}} + c_4 E_{\mathrm{SS}}^{1/2} E_{\mathrm{IN}}$$

$$\dot{E}_1 = -2\nu_0 k_1^2 E_1 + c_3 E_1^{1/2} E_{\mathrm{LS}} + c_2 \theta(E_1 - E_{1,0}) E_1^{1/2} E_{\mathrm{IN}}$$

effective wavenumbers:

$k_1 = 1,\; k_{\mathrm{LS}},\; k_{\mathrm{SS}},\; k_{\mathrm{IN}} = (k_{\min} + k_{\max})/2$

Four-scale model: upper branch $E_1 > E_{1,0}$

In steady state:

$$-2\nu_1 k_{\mathrm{IN}}^2 E_{\mathrm{IN}} = c_1 E_{\mathrm{LS}}^{1/2} E_{\mathrm{IN}} + c_2 E_1^{1/2} E_{\mathrm{IN}} \implies c_1 E_{\mathrm{LS}}^{1/2} + c_2 E_1^{1/2} = -2\nu_1 k_{\mathrm{IN}}^2 \,,$$

$$2\nu_0 k_{\mathrm{LS}}^2 E_{\mathrm{LS}} = c_1 E_{\mathrm{LS}}^{1/2} E_{\mathrm{IN}} \implies E_{\mathrm{LS}} = \left(\frac{c_1}{2\nu_0 k_{\mathrm{LS}}^2} E_{\mathrm{IN}} \right)^2 \,,$$

$$2\nu_0 k_1^2 E_1 = c_2 E_1^{1/2} E_{\mathrm{IN}} \implies E_1 = \left(\frac{c_2}{2\nu_0 k_1^2} E_{\mathrm{IN}} \right)^2 \,.$$

$$-2\nu_1 k_{\mathrm{IN}}^2 = c_1 E_{\mathrm{LS}}^{1/2} + c_2 E_1^{1/2} = \frac{\frac{c_1^2}{2k_{\mathrm{LS}}^2} + \frac{c_2^2}{2k_1^2}}{\nu_0} E_{\mathrm{IN}} \implies E_{\mathrm{IN}} = -4 \frac{\nu_1 \nu_0 k_{\mathrm{IN}}^2}{\frac{c_1^2}{k_{\mathrm{LS}}^2} + \frac{c_2^2}{k_1^2}} \,,$$

$$E_{\mathrm{IN}} \sim |\nu_1| \text{ and } E_1 \sim \nu_1^2$$

Four-scale model: lower branch $E_1 < E_{1,0}$

In steady state:

$$2\nu_1 k_{\rm IN}^2 E_{\rm IN} = -c_1 E_{\rm LS}^{1/2} E_{\rm IN} \implies E_{\rm LS} = \left(\frac{2\nu_1 k_{\rm IN}^2}{c_1}\right)^2 ,$$

$$2\nu_0 k_{\rm LS}^2 E_{\rm LS} = c_1 E_{\rm LS}^{1/2} E_{\rm IN} - c_3 E_1^{1/2} E_{\rm LS} \implies E_{\rm IN} = \frac{1}{c_1}\left(2\nu_0 k_{\rm LS}^2 + c_3 E_1^{1/2}\right) E_{\rm LS}^{1/2} ,$$

$$2\nu_0 k_1^2 E_1 = c_3 E_1^{1/2} E_{\rm LS} \implies E_1 = \left(\frac{c_3}{2\nu_0 k_1^2} E_{\rm LS}\right)^2 .$$

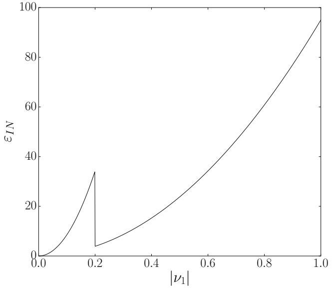
$$E_{\rm IN} = -4\frac{\nu_1 \nu_0}{c_1^2} k_{\rm IN}^2 \left(k_{\rm LS}^2 + \left(\frac{c_3 \nu_1 k_{\rm IN}^2}{c_1 \nu_0 k_1}\right)^2\right) ,$$

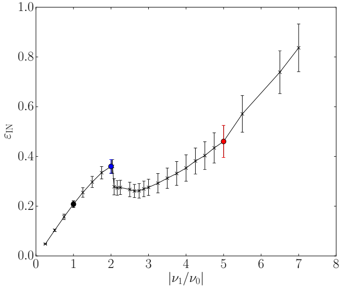
Four-scale model: transition

For $|\nu_1^{\pm}| \simeq |\nu_{1,\mathrm{crit}}|$:

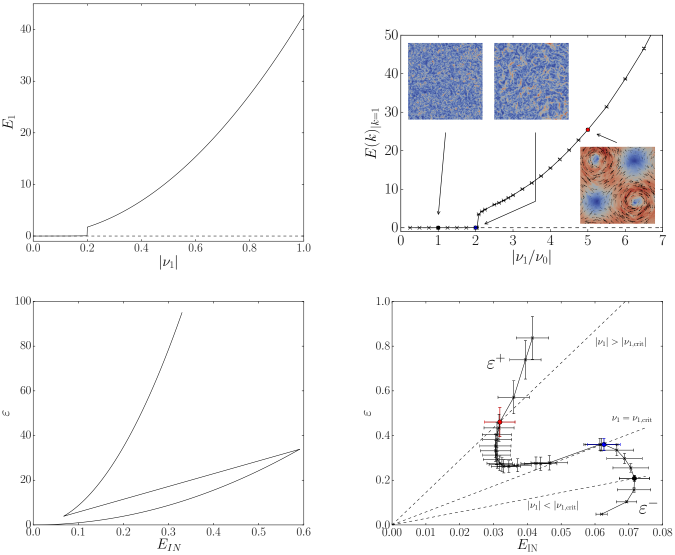
$$E_{\mathrm{IN}}^{\varepsilon_+} = 4\frac{|\nu_1^+|\nu_0 k_{\mathrm{IN}}^2}{\frac{c_1^2}{k_{\mathrm{LS}}^2}+\frac{c_2^2}{k_1^2}} < 4\frac{|\nu_1^-|\nu_0}{c_1^2}k_{\mathrm{IN}}^2\left(k_{\mathrm{LS}}^2+\left(\frac{c_3\nu_1^- k_{\mathrm{IN}}^2}{c_1\nu_0 k_1}\right)^2\right) = E_{\mathrm{IN}}^{\varepsilon_-} \,.$$

$$\varepsilon_+ = \varepsilon_{\mathrm{IN}}^+ = 2|\nu_1^+|k_{\mathrm{IN}}^2 E_{\mathrm{IN}}^{\varepsilon_+} < 2|\nu_1^-|k_{\mathrm{IN}}^2 E_{\mathrm{IN}}^{\varepsilon_-} = \varepsilon_{\mathrm{IN}}^- = \varepsilon_-$$



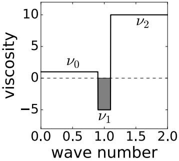


Four-scale model: transition



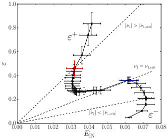
Summary

piecewise constant viscosity model in 2D:



- with weak driving, the large scales are in absolute equilibrium (no inverse energy flux)
- with strong driving, condensate forms at the largest resolved scale (inverse energy transfer)

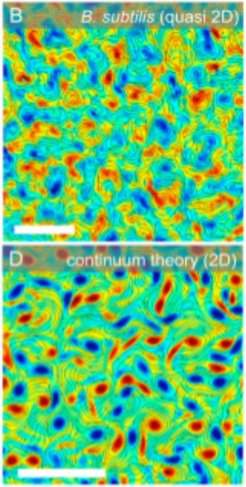
subcritical transition between two regimes (no condensate vs condensate)

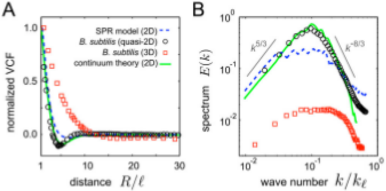


four-scale model:

direct coupling between condensate and driven scales

Comparison to experiments: Wensink et al. PNAS **109** 14308 (2012)





scale bar: $50\mu m$, domain length: $217\mu m$

Comparison with experiments

mesoscale vortices in bacterial suspensions:

Dombrowski et al PRL **93** 098103 (2004)

- size $\simeq 100\mu m$,
- speed $\simeq 100\mu m/s$,
- $\nu_{H_2O} = 10^6 \mu m^2/s$

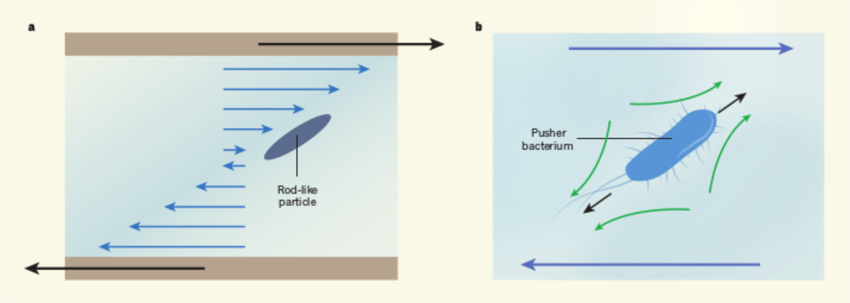
Reynolds number:
$Re = O(10^{-2})$

numerical simulations:

- Driving scale: $\pi/k_{\rm IN} \simeq 100\mu m$,
- Velocity scale: $\sqrt{E_{\rm IN}} \simeq 240\mu m/s$
- $\nu = O(10^3 \mu m^2/s)$

Reynolds number:
$Re = O(10 - 20)$

Viscosity reduction



M. C. Marchetti Nature Viewpoint **525** 37/38 (2015)

Viscosity reduction: bacterial suspensions

- viscosity reduction in suspensions of *B. subtilis*:

  Sokolov & Aranson, PRL **103** 148101 (2009)

  $O(10)$ reduction for volume fractions $8\% - 20\%$

- viscosity reduction in suspensions of *E. coli*:

  Lopéz et al, PRL **115** 028301 (2015)

  zero viscosity for volume fractions $\leqslant 1\%$

- mesoscale vortices occur for volume fractions $\geqslant 8\%$:

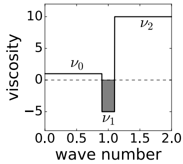
  Gachelin et al, NJP **16** 025003 (2014)

$$Re = O(10^{-1} - 1)$$

Further reduction possible?

Conclusions

piecewise constant viscosity model in 2D:



subcritical transition between two regimes

- weak driving: the large scales in absolute equilibrium
  no inverse energy flux

- strong driving: condensate at the largest scale
  inverse energy transfer

four-scale model: direct coupling to condensate

Re $O(10-100)$ larger compared to experiments

Can such Re be achieved in experiments?

Thank you