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Slide of the Seminar

Quantized Vortex Dynamics in Turbulent Quantum Fluids

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Quantized Vortex Dynamics in Turbulent Quantum Fluids

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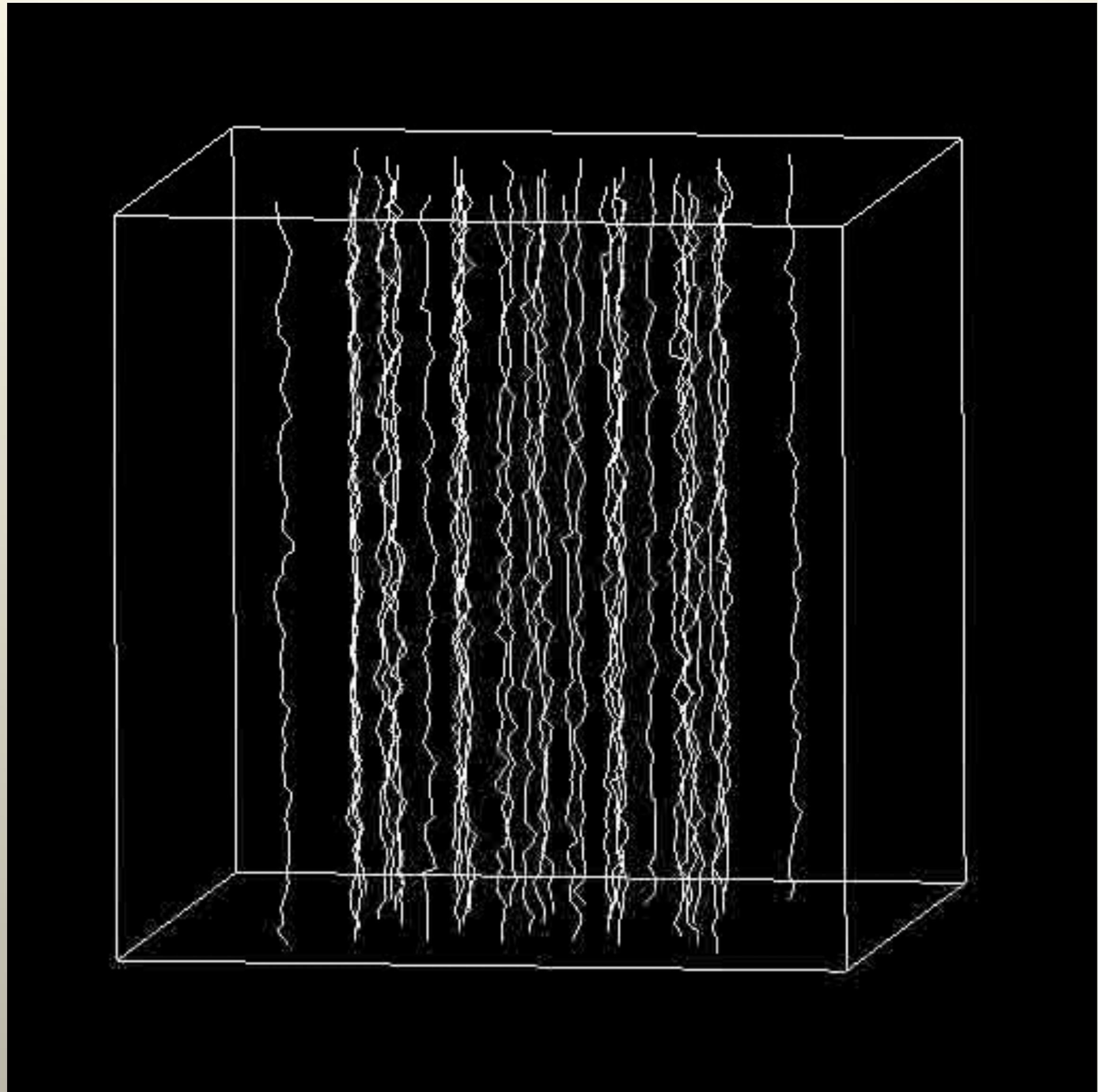
The creation of a vortex tangle: simulations



Do quantized vortices matter in turbulence?

- Experimentally and numerically the answer is `no' on the large scales: "the energy spectrum of the superflow is compatible with Kolmogorov's scaling." (M. Abid , M.E. Brachet, J. Maurer, C. Nore, P. Tabeling (1998). The two-fluid model works well.
- On small scales the dynamics of vortex lines is crucial, and strongly coupled to the velocities of the normal and super. The two-fluid model needs another order parameter.

Vinen Equation

Hall and Vinen works (1956-1958)

Vinen: Proc. R. Soc. A **238**, 204 (1956)
Proc. R. Soc. A **242**, 493 (1957)
Proc. R. Soc. A **243**, 400 (1958)
Hall : Phil. Trans. A **250**, 359 (1957)

$$d\mathcal{L}(t)/dt = \underbrace{\mathcal{P}(t)}_{production} - \underbrace{\mathcal{D}(t)}_{decay}$$

$$\mathcal{P}(t) = \alpha C_1 \mathcal{L}^{3/2} |V_{ns}| \qquad \mathcal{D}(t) = \alpha \kappa C_2 \mathcal{L}^2$$

$\mathcal{L}$ - vortex line density

$\boldsymbol{V}_{\rm ns} = \boldsymbol{V}_{\rm n} - \boldsymbol{V}_{\rm s}$ - counterflow velocity

α - mutual friction parameter

C_1 and C_2 - fitting parameters

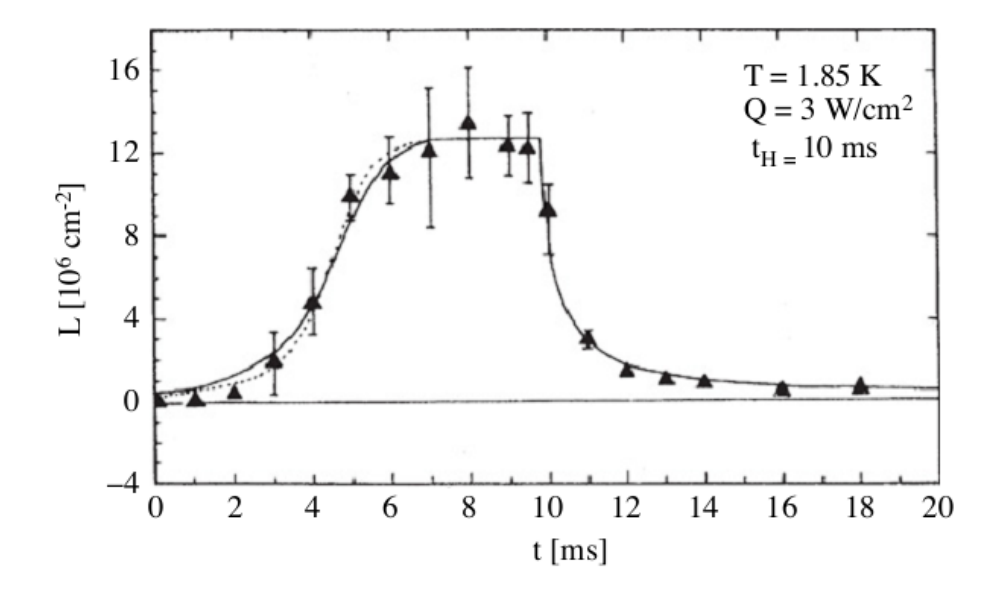
κ -quantum of circulation, for ^{4}He: $\kappa = 9.97 \times 10^{-8} cm^2/sec$

Dimensional considerations

$$\mathcal{P} \Rightarrow \mathcal{P}_{cl} = \alpha\kappa\mathcal{L}^2 F(x)$$
$$\mathcal{D} \Rightarrow \mathcal{D}_{cl} = \alpha\kappa\mathcal{L}^2 G(x)$$

$$x \equiv V_{\rm ns}^2/\kappa^2\mathcal{L}$$

$$\mathcal{P}_1 = \alpha C_1 \mathcal{L}^{3/2}|V_{ns}|$$
$$\mathcal{P}_2 = \alpha C_2 \mathcal{L} V_{ns}^2$$
$$\mathcal{P}_3 = \alpha C_3 \mathcal{L}^{1/2}|V_{ns}^3|/\kappa^2$$

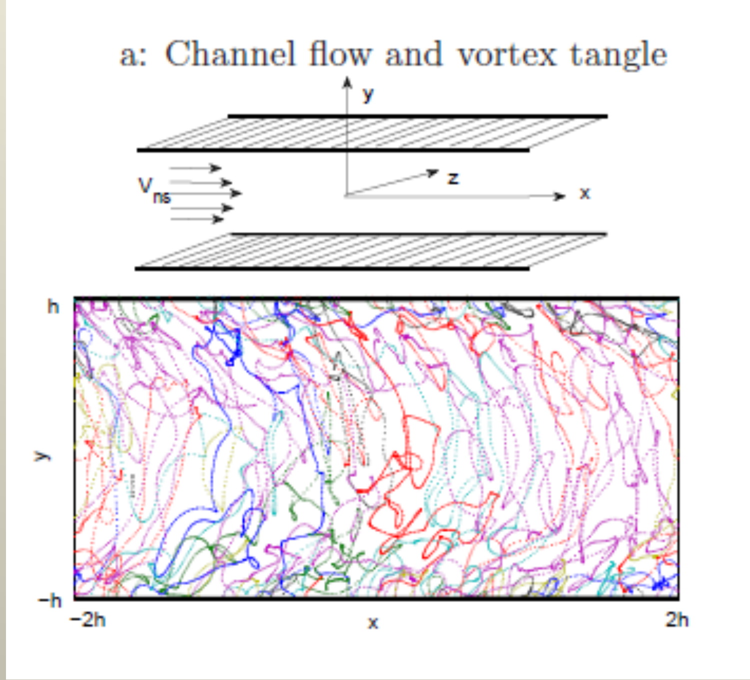


Solution of Vinen's equation with P_1 (solid line) and P_2 (dashed line) production terms

S.K. Nemirovskii et. al **PRB** 48, 1993

Comparison with homogeneous data failed to distinguish between the various dimensionally equivalent forms!

We propose therefore to consider inhomogeneous channel flows



Numerical simulation

We consider counterflow in a planar channel

- Vortex filament model using full Biot-Savart calculations
- Computational domain 0.2x0.1x0.1 cm
- Periodic boundary conditions in x, z directions
  Solid walls with slip boundary conditions
- Line resolution $\Delta\xi = 1.6 \times 10^{-3}$ cm
- Dissipative reconnection criterion
- T=1.6 K, $\alpha = 0.098, \alpha' = 0.016.$
- Two normal velocity profile types – parabolic and non-parabolic

Final result

$$\frac{\partial \mathcal{L}(\boldsymbol{r},t)}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{J}(\boldsymbol{r},t) = \mathcal{P}_3(\boldsymbol{r},t) - \mathcal{D}(\boldsymbol{r},t) \ .$$

$$\boldsymbol{J}(\boldsymbol{r},t) = -C_{\mathrm{flux}}\left(\alpha/2\kappa\right)\boldsymbol{\nabla} V_{\mathrm{ns}}^2 \ .$$

Microscopic theory

Start with the equation for a vortex line segment

K. W. Schwarz, Phys. Rev. B **18**, 245 (1978).
K. W. Schwarz, Phys. Rev. B **38**, 2398 (1988).

$$\frac{1}{\delta\xi}\frac{d\delta\xi}{dt} = \alpha V_{\rm ns}(s,t)\cdot(s' \times s'') + s'\cdot \mathbf{V}_{\rm nl}^{s}{}' - \alpha' s''\cdot \mathbf{V}_{\rm ns} \ .$$

This equation contains two temperature dependent dimensionless mutual friction parameters α and α':

You need to integrate this equation over the volume to find an equation for the vortex line density, and estimate each term in terms of velocities and the line density itself.

Can we express these integrals in terms of the velocities and the vortex line density only?

In principle the closure can be sensitive to details and may require further characteristics of the vortex tangle.

Additional candidate order parameters are
The tangle curvature and the tangle anisotropy

$$\frac{\partial \mathcal{L}(y,t)}{\partial t} + \frac{\partial J_{\rm num}(y,t)}{\partial y} = \mathcal{P}_{\rm num}(y,t) - \mathcal{D}_{\rm num}(y,t) \ , \quad (8)$$

with the following identification for the flux $J_{\rm num}$ (toward the walls), production $\mathcal{P}_{\rm num}$ and decay term $\mathcal{D}_{\rm num}$:

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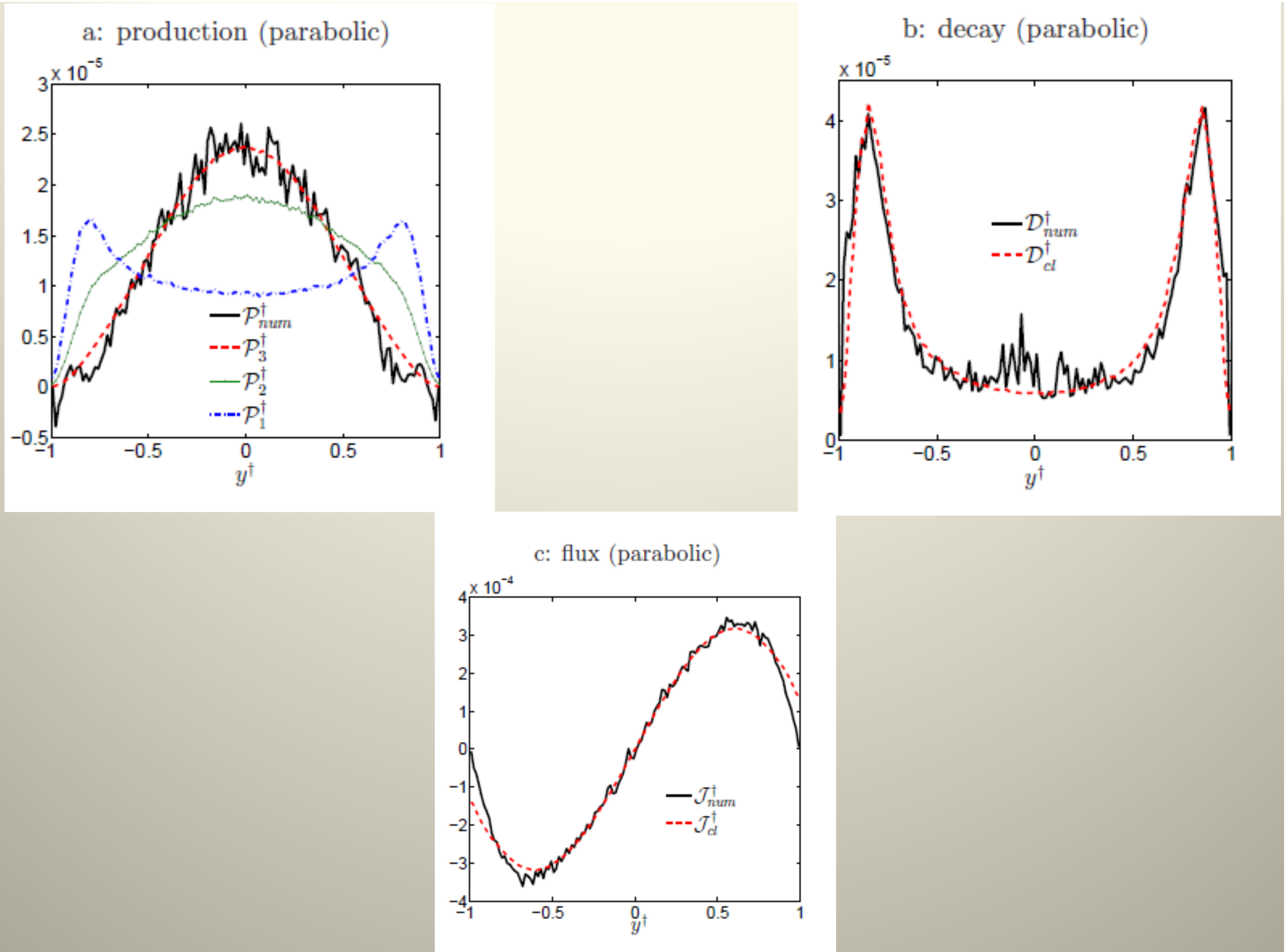
$$J_{\rm num}(y,t) = \frac{\alpha}{\Omega}\int d\xi \, V_{\rm ns,x}\, s'_z \ , \qquad (9a)_{9a}$$

$$\mathcal{P}_{\rm num}(y,t) = \frac{\alpha}{\Omega}\int d\xi \, (\boldsymbol{V}^{\rm n} - \boldsymbol{V}_0^{\rm s} - V_{\rm nl}^{\rm s}) \cdot (s' \times s'') \, , \quad (9b)_{prod}$$

$$\mathcal{D}_{\rm num}(y,t) = \frac{\alpha\beta}{\Omega}\int d\xi \, |s''|^2 = \alpha\beta\mathcal{L}\widetilde{S}^2 \ . \qquad (9c)_{dec}$$

$$\kappa\mathcal{L}(y)\,\langle s'_z\rangle_{x,z} = \frac{dV_s}{dy} \ ,$$

$$\mathcal{J}_y(y,t) \approx \frac{\alpha}{\kappa} V_{ns}\frac{dV_s}{dy} \ .$$


a: production (parabolic)
x 10^-5
$\mathcal{P}^{\dagger}_{num}$
$\mathcal{P}^{\dagger}_{3}$
$\mathcal{P}^{\dagger}_{2}$
$\mathcal{P}^{\dagger}_{1}$
$y^{\dagger}$
b: decay (parabolic)
x 10^-5
$\mathcal{D}^{\dagger}_{num}$
$\mathcal{D}^{\dagger}_{cl}$
$y^{\dagger}$
c: flux (parabolic)
x 10^-4
$\mathcal{J}^{\dagger}_{num}$
$\mathcal{J}^{\dagger}_{cl}$
$y^{\dagger}$

Are we just lucky?

- We do not know at this point.
- Much more work is necessary to draw final conclusions.

Thank you!!