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The Dynamics of Finite-sized Particles in Turbulent

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Settling, Collisions, and Coalescences: Droplets in a Turbulent Flow

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Why are we interested in particles?

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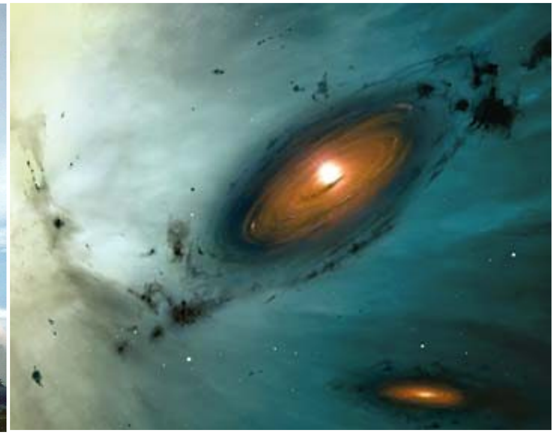
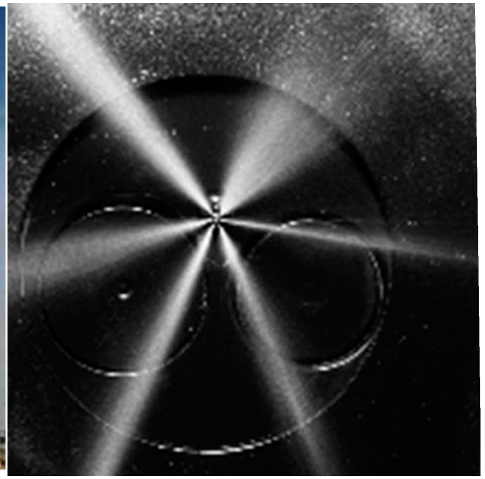


Cloud formation **Pyroclastic flows** **Planetary formation**



Pollutant dispersion **Industry** **Planktons and marine biology**

Outline

- Finite-sized, Inertial Particles: An Introduction
  - Preferential Concentration
  - Stokes Drag Model
- How Fast do Droplets Collide?
  - Validating the Stokes Drag Model
  - Extreme Events in Relative Velocities
  - Conclusions
- How Fast do Droplets Settle?
  - Settling Velocity
  - Two-particle, Small-scale Properties
    - Correlation Dimension
    - Approaching Rates
  - Conclusions
- Open Questions

Inertial Particles

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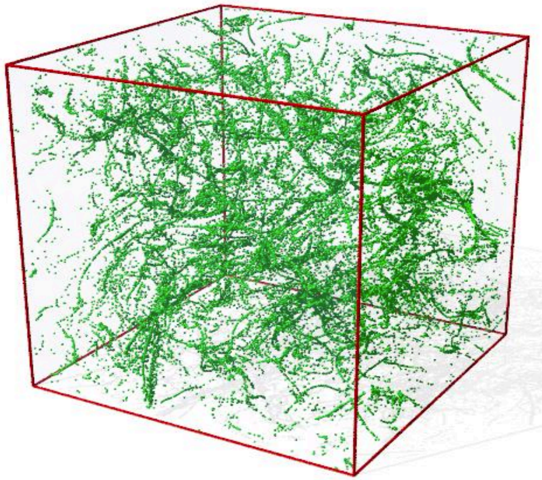
- Density different from that of the fluid.
- Finite size.
- Friction (Stokes) and other forces should be included.
- Velocity different from the underlying fluid velocity.
- Inertial particles have dissipative dynamics: Uniform contraction in phase space.

Types of Particles

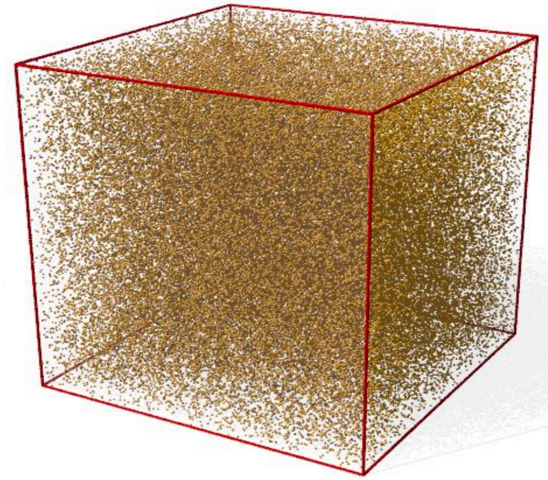
$$\boxed{\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}}$$



Light particles

$\rho_p \ll \rho_f$

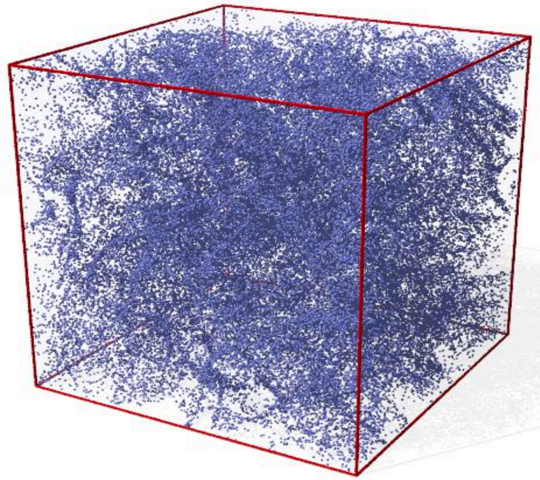
$\beta = 3$



Tracers

$\rho_p = \rho_f$

$\beta = 1$



Heavy particles

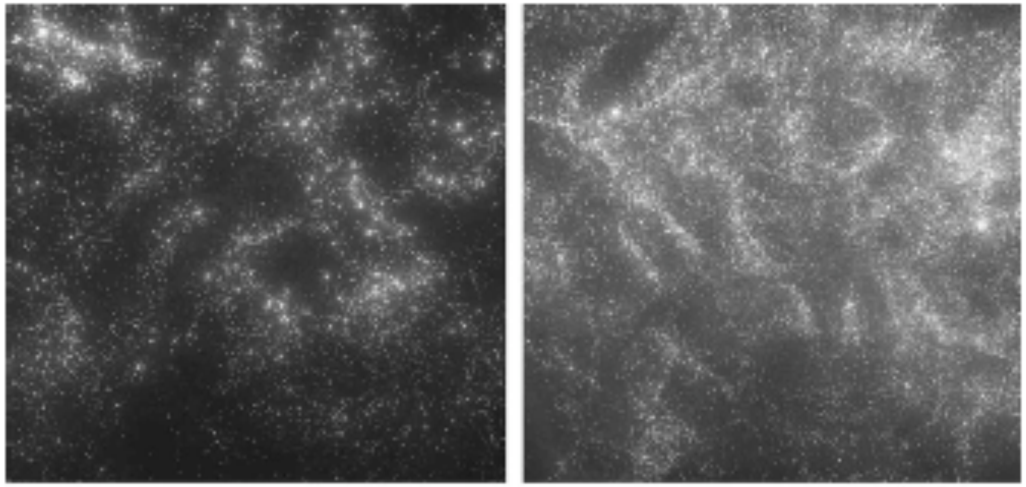
$\rho_p \gg \rho_f$

$\beta \ll 1$

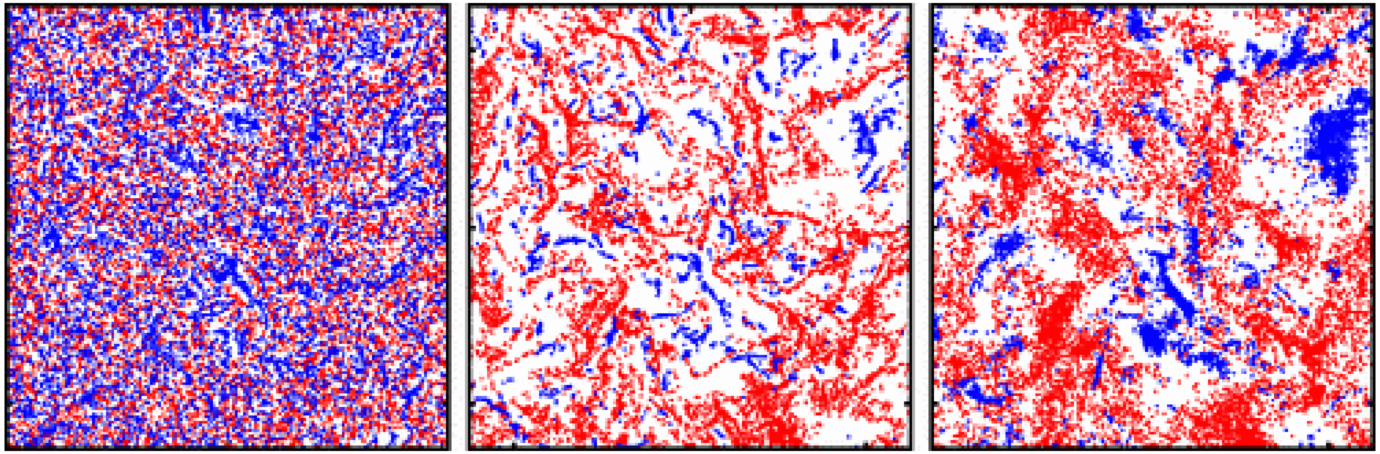
$$\boxed{\tau_p = \frac{2a^2\rho_p}{9\nu\rho_f}}$$

Effect of Inertia: Preferential Concentration

Experiments:



Simulations:



A. M. Wood, *et al.*, Int. J. Multiphase Flow, **31** (2005).
E. Calzavarini, *et al.*, Phys. Rev. Lett., **101** (2008).

Understanding Preferential Concentration

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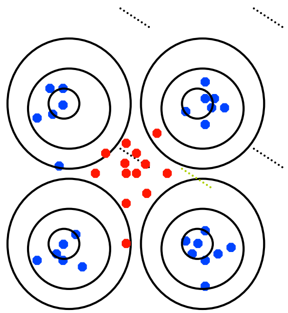
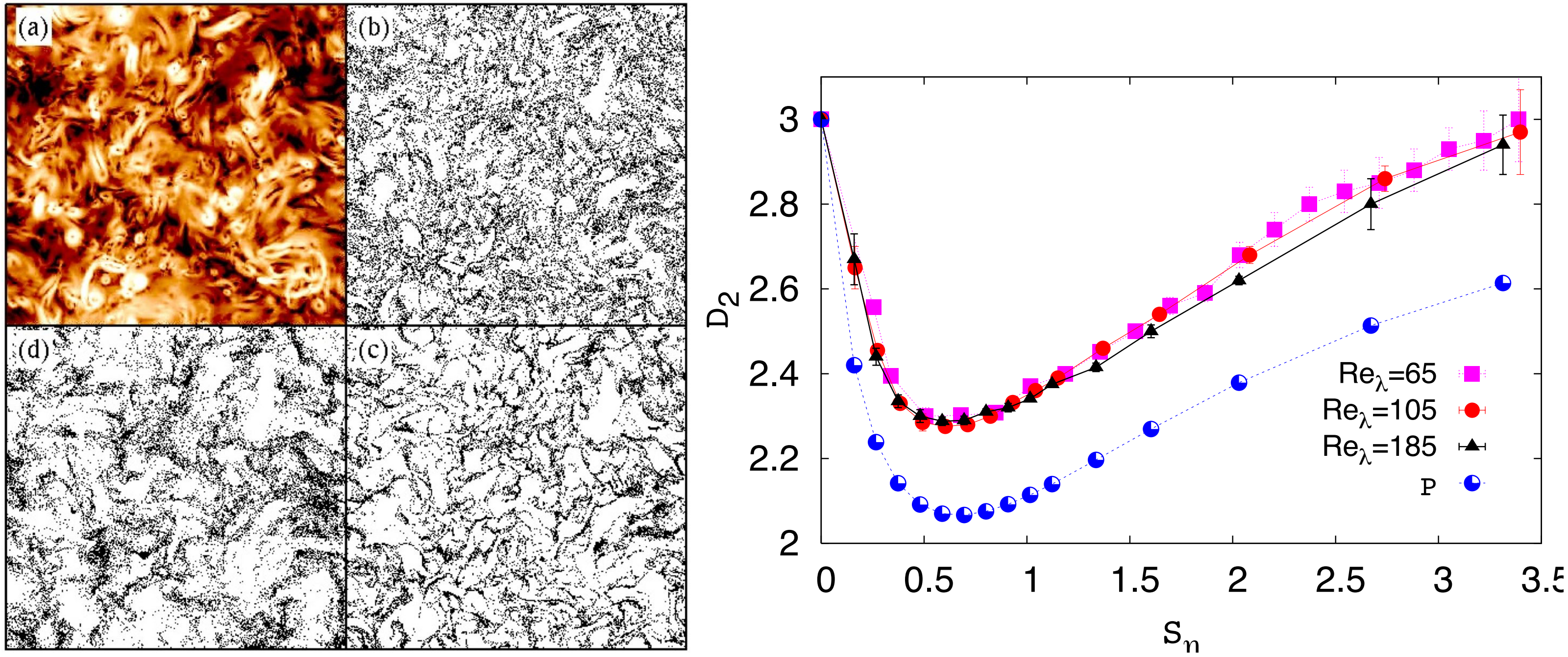
- Spatial distribution of finite-size massive particles is strongly inhomogeneous (**preferential concentration**) due to inertia.
- Qualitative understanding based on the idea that vortices act as centrifuges ejecting particles heavier than the fluid and trapping lighter ones.



- $\tau_p \to 0$: *uniform distribution*
  - $\dot{\mathbf{x}}_i = \mathbf{u}(\mathbf{x}_i, t)$; $\nabla \cdot \mathbf{u} = 0$; assumption of chaoticity.
- $\tau_p \to \infty$: *uniform distribution*
  - $\tau_f \ll \tau_p$; ballistic motion.
- Maximum clusterization is achieved for a finite value of τ_p.
- Small scale particle clusters are characterised by the correlation dimension $\mathcal{D}_2$: the probability to find two particles at a distance less than a given r is $P_2^{<}(r) \sim r^{\mathcal{D}_2}$.

Preferential Concentration: $\mathcal{D}_2$



(Left)The modulus of the pressure gradient, giving the main contribution to fluid acceleration (a). Particle positions in the same slice are shown for (b) $St_\eta = 0.16$, (c) 0.80 and (d) 3.30.
(Right) The correlation dimension $\mathcal{D}_2$ vs St_η for different R_λ.

J. Bec, *et al.*, Phys. Rev. Lett. **98** (2007).
J. Bec, *et al.*, Phys. Rev. E **87** (2013).

Single particle dynamics

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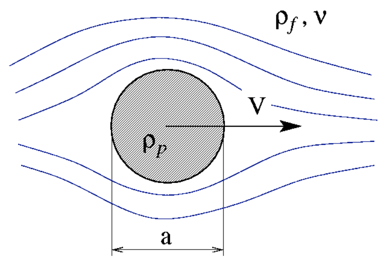
Single, passive, spherical, inertial, particle of radius a, mass m_p.

$$\rho_p \frac{d\mathbf{v}}{dt} = \rho_f \frac{D\mathbf{u}}{Dt} + (\rho_p - \rho_f)\mathbf{g} - \frac{9\nu\rho_f}{2a^2}\left(\mathbf{v} - \mathbf{u} - \frac{a^2}{6}\nabla^2\mathbf{u}\right) - \frac{\rho_f}{2}\left(\frac{d\mathbf{v}}{dt} - \frac{D}{Dt}\left[\mathbf{u} + \frac{a^2}{10}\nabla^2\mathbf{u}\right]\right)$$



$$\boxed{\frac{a(u-V)}{\nu} \ll 1 \qquad a \ll \eta}$$

$\rho_f \frac{D\mathbf{u}}{Dt} \longrightarrow$ force by the undisturbed flow;	$\frac{9\nu\rho_f}{2a^2}\left(\mathbf{v} - \mathbf{u} - \frac{a^2}{6}\nabla^2\mathbf{u}\right) \longrightarrow$ Stokes drag;
$(\rho_p - \rho_f)\mathbf{g} \longrightarrow$ buoyancy;	$\frac{\rho_f}{2}\left(\frac{d\mathbf{v}}{dt} - \frac{D}{Dt}\left[\mathbf{u} + \frac{a^2}{10}\nabla^2\mathbf{u}\right]\right) \longrightarrow$ added mass

$$D\mathbf{u}/Dt = \partial\mathbf{u}/\partial t + (\mathbf{u}\cdot\nabla)\mathbf{u}$$

$$d\mathbf{u}/dt = \partial\mathbf{u}/\partial t + (\mathbf{v}\cdot\nabla)\mathbf{u}$$

M. R. Maxey & J. J. Riley, Phys. Fluids **26**, 883 (1983).

Stokes Drag Model

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Setting:

- small sized particles;
- dilute suspensions;
- passive particles.

Simplifications:

- The Faxen correction $a^2\nabla^2\mathbf{u} \approx \mathcal{O}(a^2 u/L) \ll 1$.
- $\frac{D\mathbf{u}}{Dt} \approx \frac{d\mathbf{v}}{dt}$
- Buoyancy effects negligible.

Working Equations:

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= \mathbf{v}; \\ \frac{d\mathbf{v}}{dt} &= -\frac{\mathbf{v}-\mathbf{u}}{\tau_p} + \beta\frac{D\mathbf{u}}{Dt}. \end{aligned}$$

Stokes Drag Model

Setting:

- small sized particles;
- dilute suspensions;
- passive particles.

Simplifications:

- The Faxen correction $a^2\nabla^2\mathbf{u} \approx \mathcal{O}(a^2u/L) \ll 1$.
- $\frac{D\mathbf{u}}{Dt} \approx \frac{d\mathbf{v}}{dt}$
- Buoyancy effects negligible.

Working Equations (for heavy particles):

$$\begin{aligned}\frac{d\mathbf{x}}{dt} &= \mathbf{v};\\ \frac{d\mathbf{v}}{dt} &= -\frac{\mathbf{v}-\mathbf{u}}{\tau_p}.\end{aligned}$$

Model for Heavy Particles in Turbulent Flows

- **The Fluid**
  - The fluid velocity $\mathbf{u}$ is a solution of the incompressible Navier–Stokes equation and obtained via pseudo-spectral, direct numerical simulations.
  - Statistically steady, homogeneous, isotropic turbulence is maintained by a large-scale forcing.
- **The Particles**
  - Particles are much smaller than the Kolmogorov scale, much heavier than the surrounding fluid, and with a small Reynolds number associated to their slip velocity.
  - Non-dimensionless numbers:
    - *Stokes number*: $St = \tau_{\mathrm{p}}/\tau_\eta$, where $\tau_\eta = \sqrt{\nu/\varepsilon}$.
    - *Froude number*: $Fr = a_\eta/g$, where $a_\eta = \varepsilon^{3/4}/\nu^{1/4}$.

The Model: Equations

- **The Fluid**
  - The incompressible, forced Navier–Stokes equation:

$$\begin{aligned} \partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} &= -\nabla p + \nu\nabla^2\mathbf{u} + \mathbf{f}; \\ \nabla\cdot\mathbf{u} &= 0. \end{aligned}$$

    - ν is the fluid kinematic viscosity and $\mathbf{f}$ a large scale forcing.
- **The Particles**
  - Stokes drag and gravity:

$$\begin{aligned} \frac{d\mathbf{x}_{\rm p}}{dt} &= \mathbf{v}_{\rm p}; \\ \frac{d\mathbf{v}_{\rm p}}{dt} &= -\frac{1}{\tau_{\rm p}}\left[\mathbf{v}_{\rm p} - \mathbf{u}(\mathbf{x}_{\rm p}, t)\right] + \mathbf{g}. \end{aligned}$$

    - $\mathbf{u}(\mathbf{x}_{\rm p}, t)$ is evaluated by linear interpolation.

How Fast do Droplets Collide?

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Extreme fluctuations of the relative velocities between droplets in turbulent airflow

- Experiments (*with* Ewe-Wei Saw, Gregory P. Bewley, and Eberhard Bodenschatz, Göttingen, Germany)
- Theory and Direct Numerical Simulations (*with* Jérémie Bec, Nice, France)

Ewe-Wei Saw, Gregory P. Bewley, Eberhard Bodenschatz, Samriddhi Sankar Ray, and Jérémie Bec
Physics of Fluids Letters, 26, 111702 (2014).

Introduction

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- In warm clouds, turbulence in the airflow enhances the collision rate of the water droplets.
- It thus influences the evolution of droplet sizes and the timescale for rain formation.
- Two mechanisms are at play:
    - preferential concentration;
    - very large approach velocities explained in terms of the *sling effect* and the subsequent formation of *caustics*.
- Open question regarding the coalescence rate of droplets.
    - Collisions that are too violent can cause particle fragmentation.
- Developing an understanding:
    - Experiments
    - Theory
    - Direct Numerical Simulations

G. Falkovich, *et al*, Nature **419**, (2002).
R. Shaw, Ann. Rev. Fluid Mech. **35** (2003).
E.-W. Saw, *et al.*, Phys. Rev. Lett. **100** (2008).
M. Wilkinson, *et al*, Phys. Rev. Lett. **97** (2006).
E. Balkovsky, *et al.*, Phys. Rev. Lett. **86** (2001).
J. Bec, *et al*, Phys. Rev. Lett. **98** (2007).
G. P. Bewley, *et al.*, New J. Phys. **15** (2013).
G. Falkovich & A. Pumir, J. Atmos. Sci. **64** (2007).

Questions

What is the distribution of relative velocities of colliding droplets in a turbulent airflow?

Is the linear Stokes drag model valid?

We perform direct numerical simulations and experiments, with matching parameters, of droplets in a turbulent flow to answer these two questions.

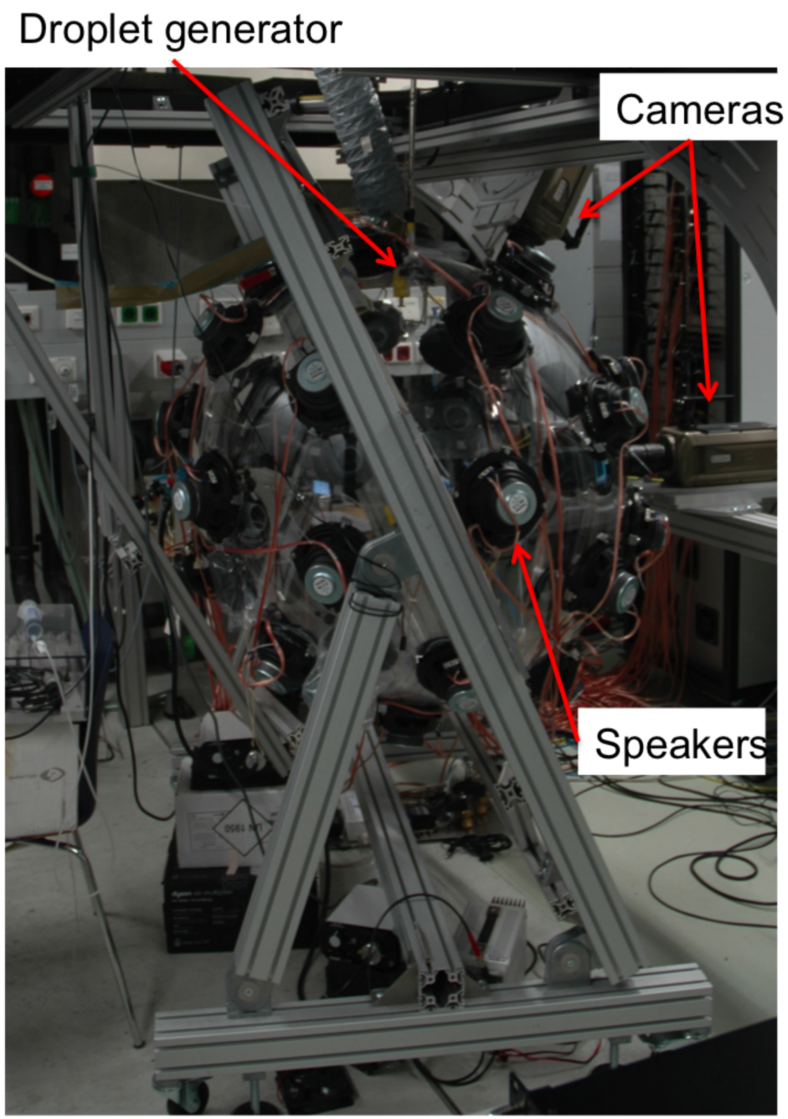
Saw, Bewley, Bodenschatz, Ray, and Bec, Phys. Fluids Lett., **26**, 111702, (2014).

Experiment

- Homogeneous and isotropic turbulent flows are generated in a 1 *m*-diameter acrylic sphere by 32 randomly pulsating jets in a region of about 10 *cm* at the center.
- We ran the experiment for $R_\lambda = 160$ ($\eta = 300\mu m$), 170 ($\eta = 230\mu m$), and 190 ($\eta = 180\mu m$).
- Droplets are produced with a spinning disc device that eject bi-disperse drops with diameters $6.8\mu m$ and $19\mu m$.
- The motion of the droplets are measured by an imaging of their shadows projected by white light sources.
- The three-dimensional positions of the droplets are determined by stereoscopic Lagrangian Particle Tracking.

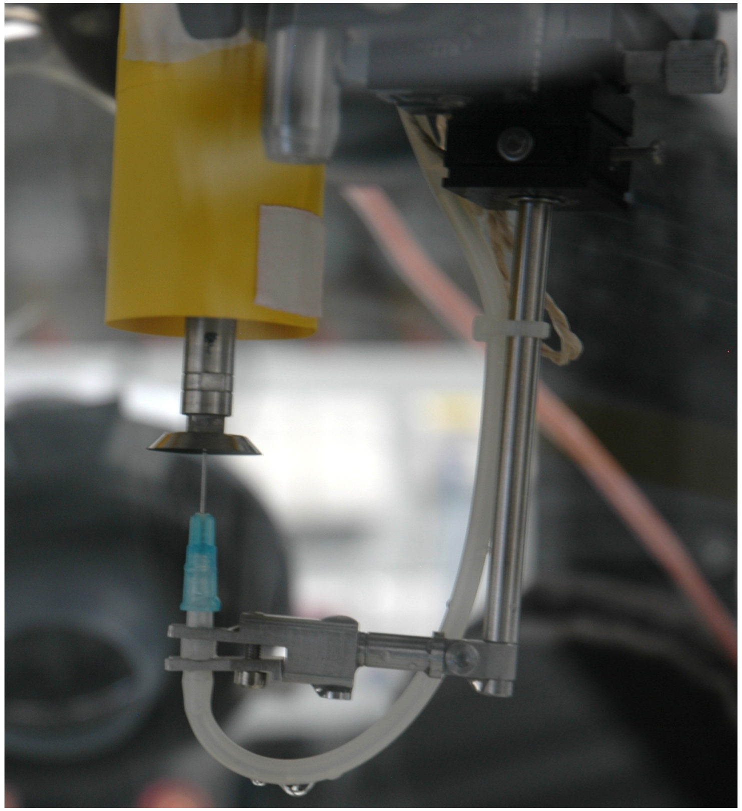
G. P. Bewley, *et al.*, New J. Phys. **15** (2013).
W. H. Walton & W. C. Prewett, Proc. Phys. Soc. B **62** (1949).
K. Chang, *et al.*, J. Fluid Mech. **692** (2012).
N. Ouellette, *et al.*, New J. Phys. **8** (2006).

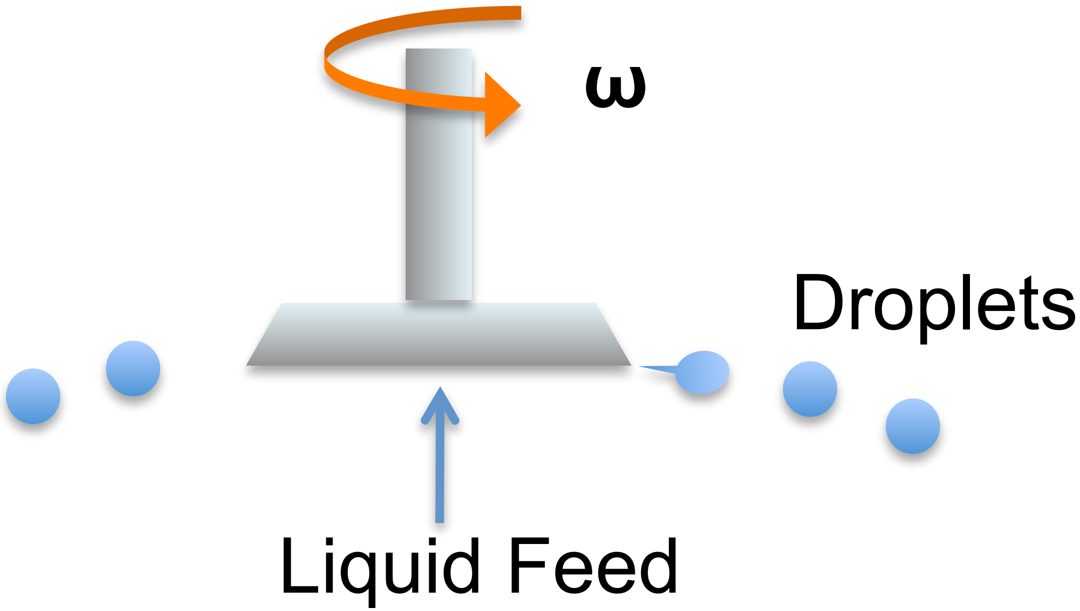
Experiment: Soccer Ball



Max Planck Institute for Dynamics and Self-Organization (MPIDS), Göttingen, Germany

Experiment: Droplet Generator





Direct Numerical Simulations (DNS)

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- **The Fluid**
  - The incompressible, forced Navier–Stokes equation:

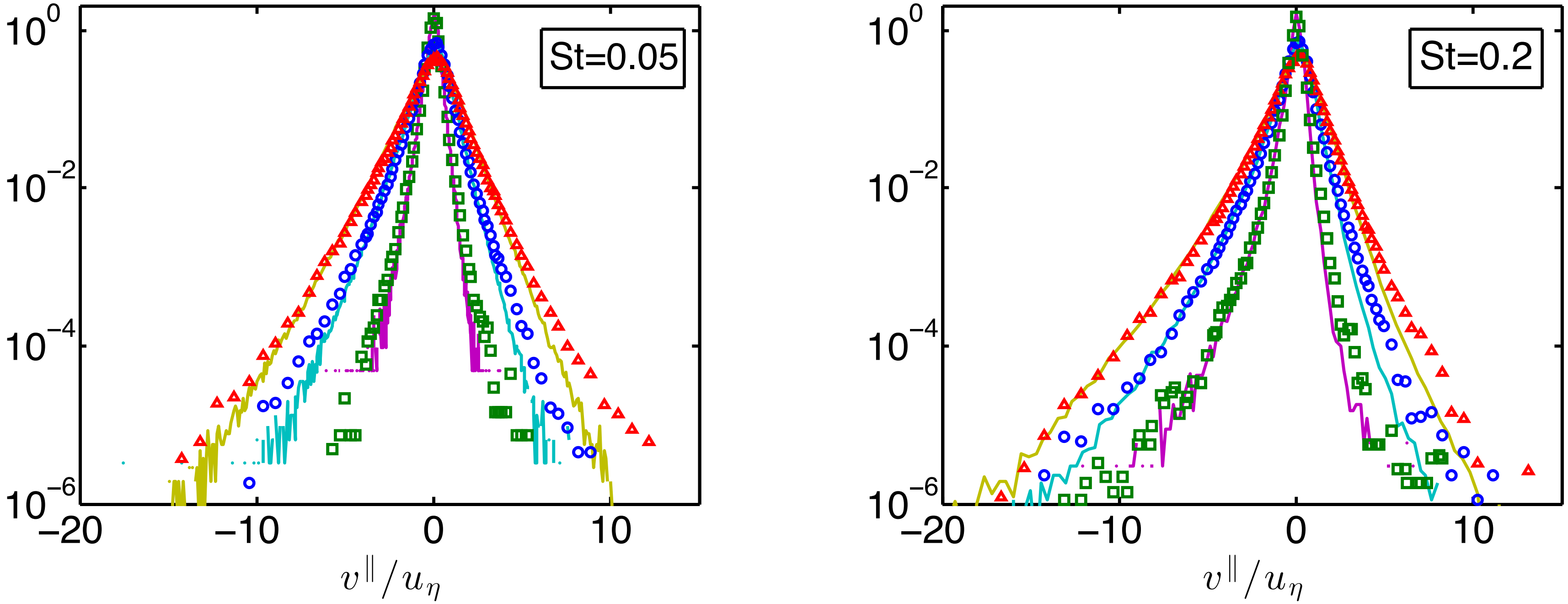
$$\begin{aligned}\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} &= -\nabla p + \nu\nabla^2\mathbf{u} + \mathbf{f};\\ \nabla\cdot\mathbf{u} &= 0.\end{aligned}$$

    - Pseudo-spectral parallel solver for the fluid velocity with 512^3 grid points and $\nu = 1.5\times10^{-4}$ ($R_\lambda = 180$).
- **The Particles**
  - Stokes drag and gravity:

$$\begin{aligned}\frac{d\mathbf{x}_\mathrm{p}}{dt} &= \mathbf{v}_\mathrm{p};\\ \frac{d\mathbf{v}_\mathrm{p}}{dt} &= -\frac{1}{\tau_\mathrm{p}}\left[\mathbf{v}_\mathrm{p} - \mathbf{u}(\mathbf{x}_\mathrm{p}, t)\right] + \mathbf{g}.\end{aligned}$$

    - $\mathbf{u}(\mathbf{x}_\mathrm{p}, t)$ is evaluated by linear interpolation.
    - Number of particles $N_p = 10^8$.

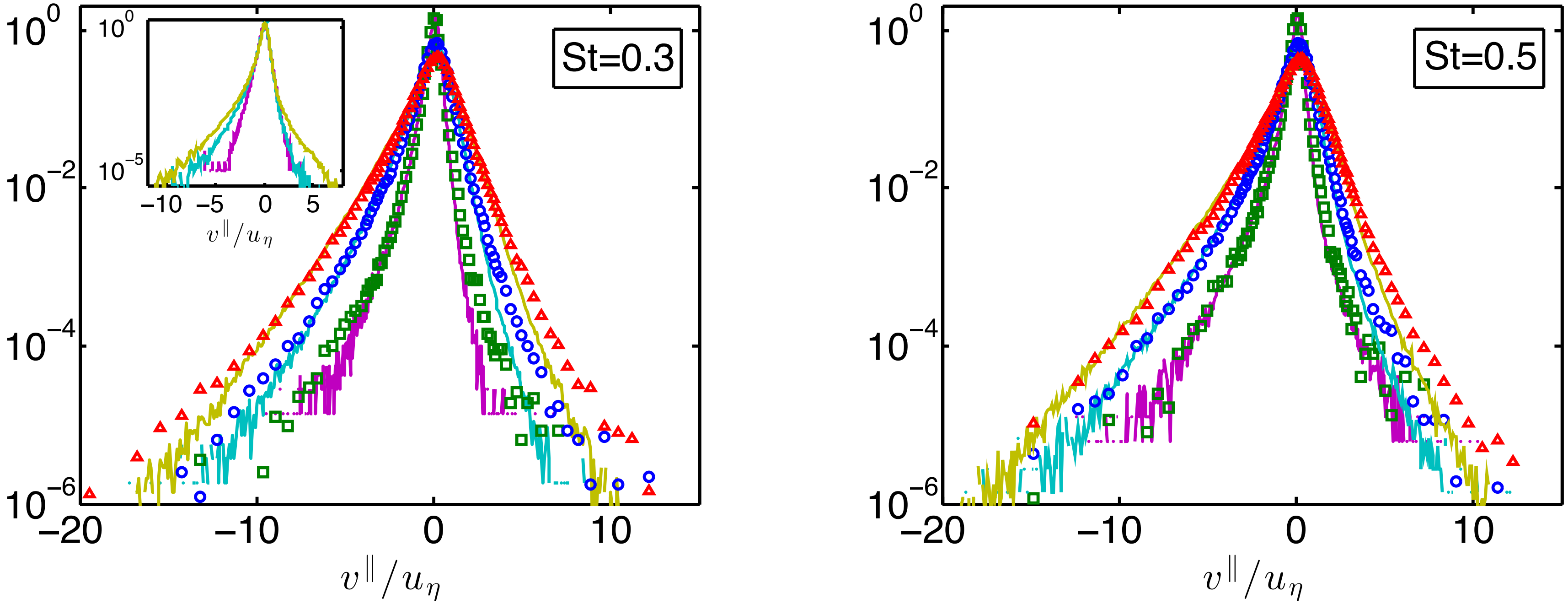
Relative Velocity: PDF



Probability distribution functions of the longitudinal velocity differences conditioned on different separations r for particles with (left) $St = 0.05$ ($St_{DNS} = 0.05$, $St_{\mathrm{experiment}} = 0.04$) and (right) $St = 0.2$ ($St_{DNS} = 0.24$, $St_{\mathrm{experiment}} = 0.19$). The symbols are the experimental data and solid lines are the DNS data. In all panels, for the experiment (DNS) data, squares (purple) correspond to $r = 1 - 1.6\eta$, circles (cyan) to $r = 3 - 3.6\eta$, and triangles (gold) to $r = 5 - 5.6\eta$.

Saw, Bewley, Bodenschatz, Ray, and Bec, Phys. Fluids Lett., **26**, 111702, (2014).

Relative Velocity: PDF



Probability distribution functions of the longitudinal velocity differences conditioned on different separations r for particles with (left) $St = 0.3$ and (right) $St = 0.5$. The symbols are the experimental data and solid lines are the DNS data. In all panels, for the experiment (DNS) data, squares (purple) correspond to $r = 1 - 1.6\eta$, circles (cyan) to $r = 3 - 3.6\eta$, and triangles (gold) to $r = 5 - 5.6\eta$. The inset shows the variation with respect to St, with the separation fixed to $r = 1 - 1.6\eta$. From the bottom to the top curve, $St = 0.05$, 0.3, 0.5.

Saw, Bewley, Bodenschatz, Ray, and Bec, Phys. Fluids Lett., **26**, 111702, (2014).

Validity of the Stokes Drag Model?

- Quantitatively, we found the differences between experiments and simulations to be less than about 15% in the core of the distributions.
- Similarly, we found excellent agreement in the tails of the distributions, but only for the largest Stokes number ($St = 0.5$), the smallest scale ($r < 2\,\eta$), and for the left side of the distributions corresponding to approaching particle pairs.
- In other cases, the experimental tails of the PDFs increasingly deviate from the simulated ones as one moves to higher relative velocities.
- The discrepancy is larger in the right tails, corresponding to separating pairs, where in the worst case the experimental data is about 5 times above the simulated data.
- In the left tails, the discrepancy is less severe, but worsens with decreasing St, so that the largest discrepancy is a factor of two.

Possible Causes of Discrepancy

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- Effects beyond linear Stokes drag maybe at play.
  - The history term may play an important role, which, given the experimental conditions, is the first subdominant correction.
- Measurement uncertainty.
  - We characterized the measurement noise and add it to the DNS data.
  - Estimated ε, in the experiment by applying the same method to the DNS data to obtain a 5% agreement.
  - We ruled out the possibility of a Reynolds number effect by comparing DNS data at increasing Reynolds numbers.
  - Inaccuracy of ν in the experiment was checked by reprocessing the experimental data with a modified ν ($\pm 30\%$) with no clear improvement.
- No clear explanation for the discrepancies.
- The influences of nonlinear forces, hydrodynamic interactions, and non-universal turbulence statistics merit further study.

M. R. Maxey & J. J. Riley, Physics of Fluids **26**, 883 (1983).
A. Daitche & T. Tél, Phys. Rev. Lett. **107**, (2011).

Velocities of Colliding Droplets

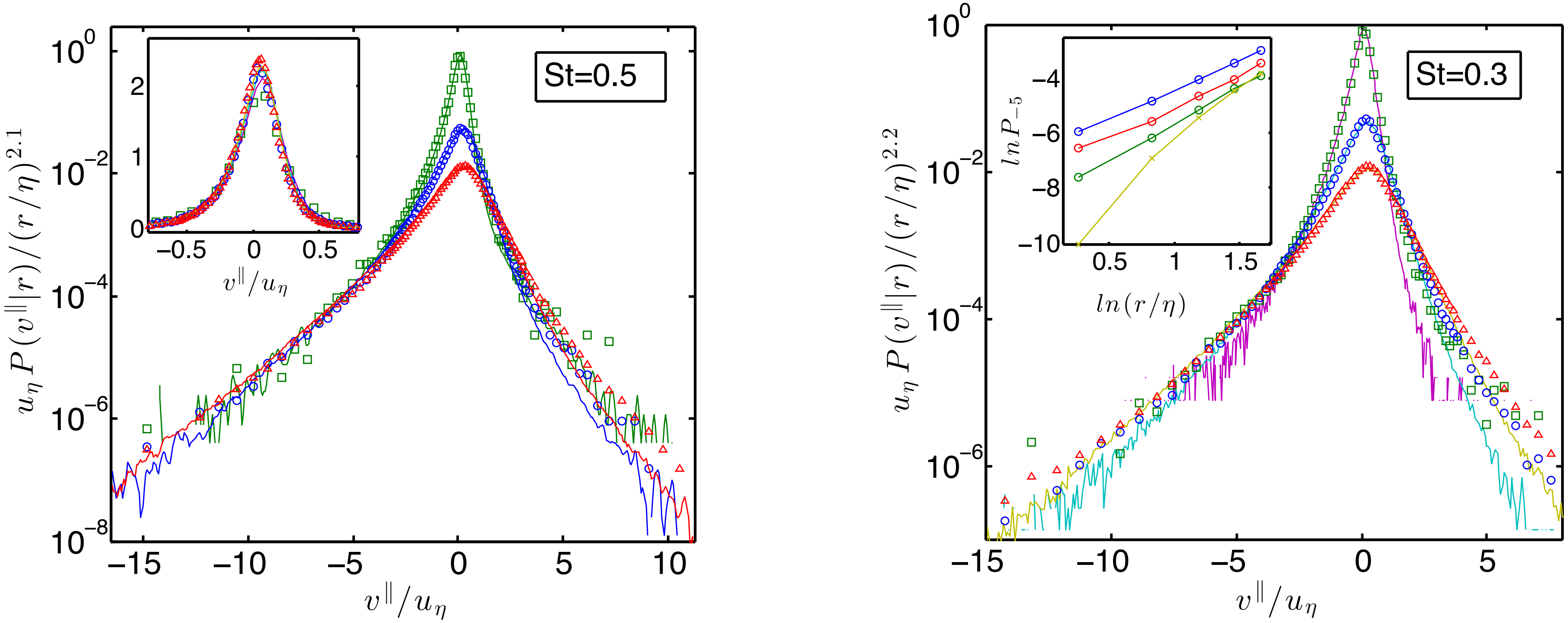
- There is general agreement in the trends and shapes of the distributions.
- All the distributions can be approximated by stretched-exponentials whose concavity increases with increasing St and decreasing r.
- This is qualitatively consistent with what is known about the velocity distributions of fluid particles.
- Earlier prediction of *compressed* exponential distributions for very large St not observed.
    - The distributions we measure are stretched rather than compressed, and the implication is that the large St limit taken in the theory does not accurately describe the intermediate St dynamics studied here.

P. Kailasnath, *et al.*, Phys. Rev. Lett. **68** (1992).
K. Gustavsson, *et al.*, Phys. Rev. Lett. **101** (2008).

Scale-dependence of Relative Velocities

- To understand droplet collision-coalescence in clouds, one needs to characterize droplet relative velocities at contact.
- Hence it is important to understand how droplet relative velocities scale with vanishing r.
- Experimental and DNS data collapse at large negative values of $v^{\|}$ when the PDF is rescaled by r^{β} with $\beta \approx 2.1$.
- Such collapse indicates that the distribution of approaching velocities takes the form $p(v^{\|} \mid r) \simeq r^{\beta(St)} \phi(v^{\|})$ at sufficiently small separations and large velocities.
- This behavior is expected to extend down to separations of the order of the particle size and hence should describe the distribution of violent impact velocities between particles.

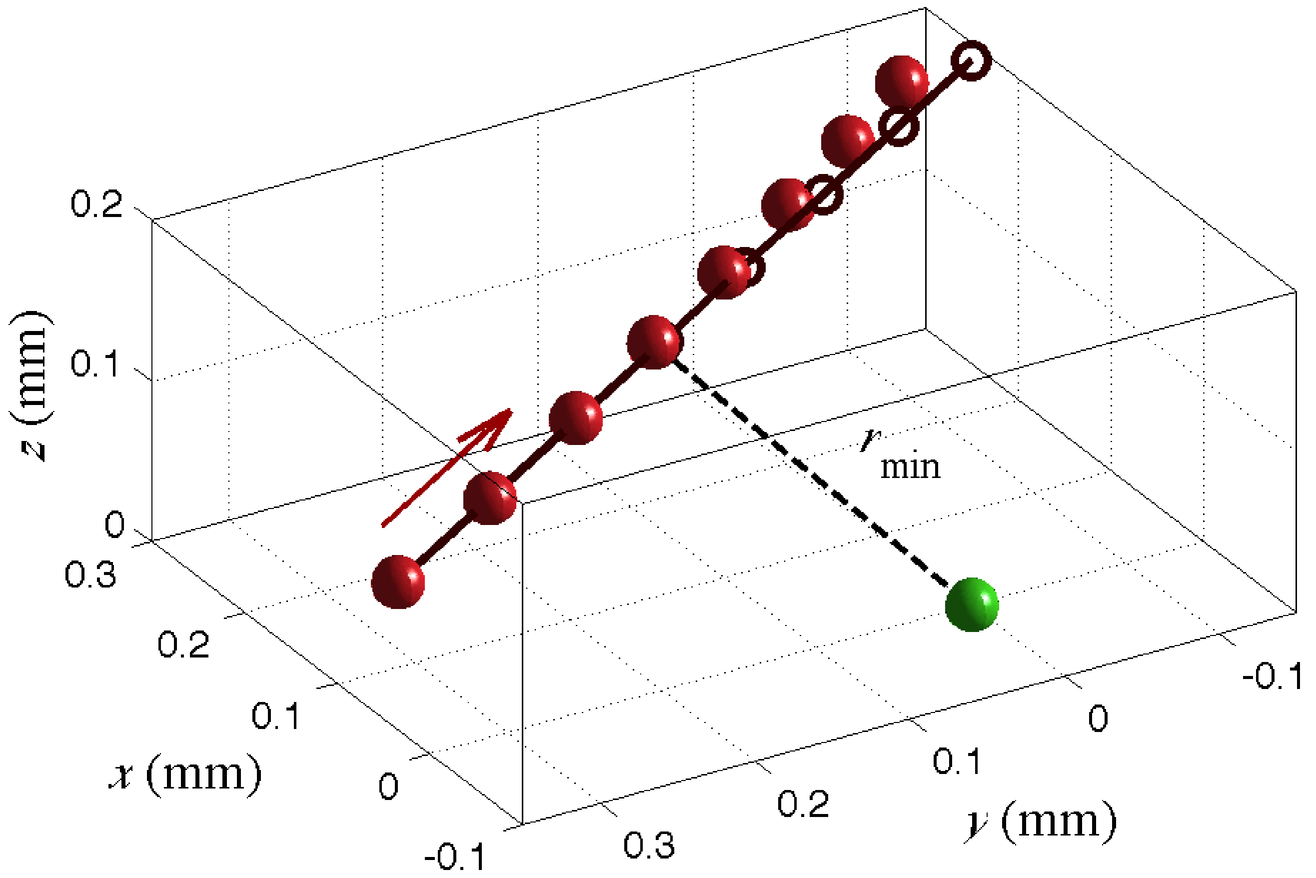
Relative Velocity: Rescaled PDF



Rescaled probability distributions of the longitudinal velocity difference conditioned on different separations r for both the experimental (symbols) and DNS (solid lines) data for (left) $St = 0.5$ with $\beta = 2.1$ and (right) $St = 0.3$, with $\beta = 2.2$. Green corresponds to $r = 1 - 1.6\eta$, blue to $r = 3 - 3.6\eta$, and red to $r = 5 - 5.6\eta$. Inset (left): r-scaling of the distribution bulk; collapse is attained by $r \times p(v^\parallel|r)$ and $(1/r) \times v^\parallel/u_\eta$. Inset (right): plots of $\ln[\Pr(v^\parallel/u_\eta < 5\,|r)]$ (denoted as lnP_{-5}) versus $\ln(r/\eta)$ for different St from the experiment. Unambiguous values of β could not be obtained at such low St.

Saw, Bewley, Bodenschatz, Ray, and Bec, Phys. Fluids Lett., **26**, 111702, (2014).

Reconstructing Particle Tracks



Reconstructing tracks of individual particles.

Conclusions

- We evaluated the accuracy of the Stokes drag model by comparing results from DNS with experimental measurements.
- For relative velocities, the DNS matched all qualitative trends of the experiments.
- Quantitative agreements for inertia-dominated regimes.
- Discrepancies found in some regimes.
- No trivial explanations for such discrepancies.
  - Corrections to the Stokes drag model.
  - Hydrodynamic interactions between particles.
  - Small-scale, non-universality of the turbulence.
- For inertial particles, at dissipative scales of turbulence:

$$p(v^{\parallel} \mid r) \simeq r^{\beta(St)}\, \phi(v^{\parallel}).$$

How Fast do Droplets Settle Under Gravity?

Gravity-driven enhancement of heavy particle clustering in turbulent flow

- Direct Numerical Simulations
- Theory: Asymptotic Expansion

Jérémie Bec, Holger Homann, and Samriddhi Sankar Ray.
Physical Review Letters 112, 184501 (2014)

Introduction

- Many industrial, atmospheric, and astrophysical phenomena involves the interactions between small solid particles suspended in a turbulent carrier flow.
- Two main effects:
  - a viscous drag on the particles (*dominant for small particles*);
  - external forces, such as gravity, on the particles (*dominant for large particles*).
- Standard modelling treats these two limits separately and often fails at the interface.
  - Example: the rate at which rain is triggered in warm clouds.
- An improvement might be to combine the effects of turbulence and gravity.

G. Falkovich, *et al*, Nature **419**, (2002).
W. Grabowski & L.-P. Wang, Annu. Rev. Fluid Mech. **45**, (2013).

Introduction

- In turbulent flows, there is an increase of the terminal velocity of heavy particles.
- This phenomenon is mostly understood on qualitative grounds and has been quantified only in model flows.
- Very little is known on the effect of gravitational settling on two-particle statistics.
- Fundamental theoretical and numerical studies of the clustering of particle pairs and of the enhancement of collisions due to inertia usually neglect gravity.

M. Maxey, J. Fluid Mech. **174**, (1987).
L.-P. Wang & M. Maxey, J. Fluid Mech. **256**, 27 (1993).
E. Balkovsky, *et al*, Phys. Rev. Lett. **86**, (2001).
J. Davila & J. Hunt, J. Fluid Mech. **440**, (2001).
M. Wilkinson, *et al*, Phys. Rev. Lett. **97**, (2006).
O. Ayala, *et al*, New J. Phys. **10**, (2008).
J. Bec, *et al*, Phys. Rev. Lett. **98**, (2007).
J. Bec, *et al*, Fluid Mech. **646**, (2010).

Question

What is the interplay between turbulence, gravity, and particle sizes?

Important for fluid dynamics and non-equilibrium statistical physics.

Bec, Homann, and Ray, Phys. Rev. Lett. **112**, 184501 (2014).

Our Approach

- We combine direct numerical simulations with theoretical results based on *standard* asymptotic analysis.
- We make a systematic study of the dynamical and statistical properties of particles as a function of
    - the level of turbulence of the carrier flow (Reynolds number);
    - the inertia of the particles (Stokes number);
    - the ratio between the turbulent accelerations and gravity (Froude number).

The Model

- **The Fluid**
  - The fluid velocity $\mathbf{u}$ is a solution of the incompressible Navier–Stokes equation and obtained via pseudo-spectral, direct numerical simulations.
  - Statistically steady, homogeneous, isotropic turbulence is maintained by a large-scale forcing.
- **The Particles**
  - Particles are much smaller than the Kolmogorov scale, much heavier than the surrounding fluid, and with a small Reynolds number associated to their slip velocity.
  - Non-dimensionless numbers:
    - *Stokes number*: $St = \tau_{\mathrm{p}}/\tau_\eta$, where $\tau_\eta = \sqrt{\nu/\varepsilon}$.
    - *Froude number*: $Fr = a_\eta/g$, where $a_\eta = \varepsilon^{3/4}/\nu^{1/4}$.
  - We use 10 different Stokes numbers and 5 different values of the Froude number

The Model: Equations

- **The Fluid**
  - The incompressible, forced Navier–Stokes equation:

$$\begin{aligned}\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} &= -\nabla p + \nu\nabla^2\mathbf{u} + \mathbf{f};\\ \nabla\cdot\mathbf{u} &= 0.\end{aligned}$$

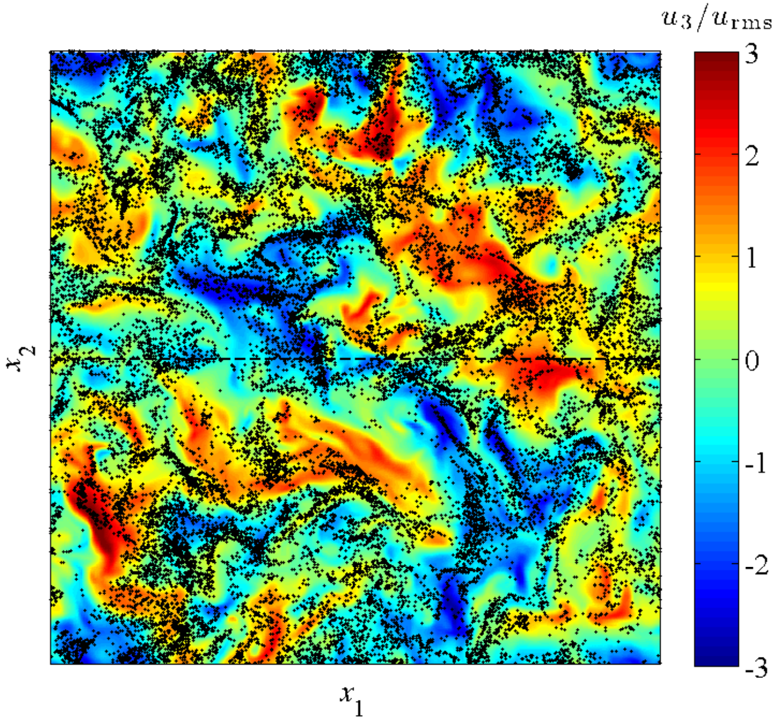
    - ν is the fluid kinematic viscosity and $\mathbf{f}$ a large scale forcing.
- **The Particles**
  - Stokes drag and gravity:

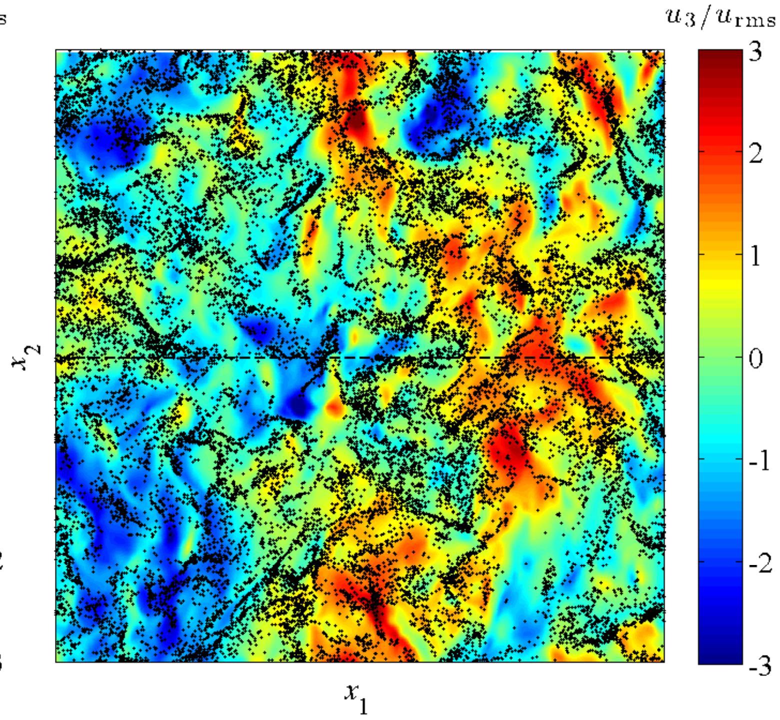
$$\begin{aligned}\frac{d\mathbf{x}_\mathrm{p}}{dt} &= \mathbf{v}_\mathrm{p};\\ \frac{d\mathbf{v}_\mathrm{p}}{dt} &= -\frac{1}{\tau_\mathrm{p}}\left[\mathbf{v}_\mathrm{p} - \mathbf{u}(\mathbf{x}_\mathrm{p}, t)\right] + \mathbf{g}.\end{aligned}$$

    - $\mathbf{u}(\mathbf{x}_\mathrm{p}, t)$ is evaluated by linear interpolation.

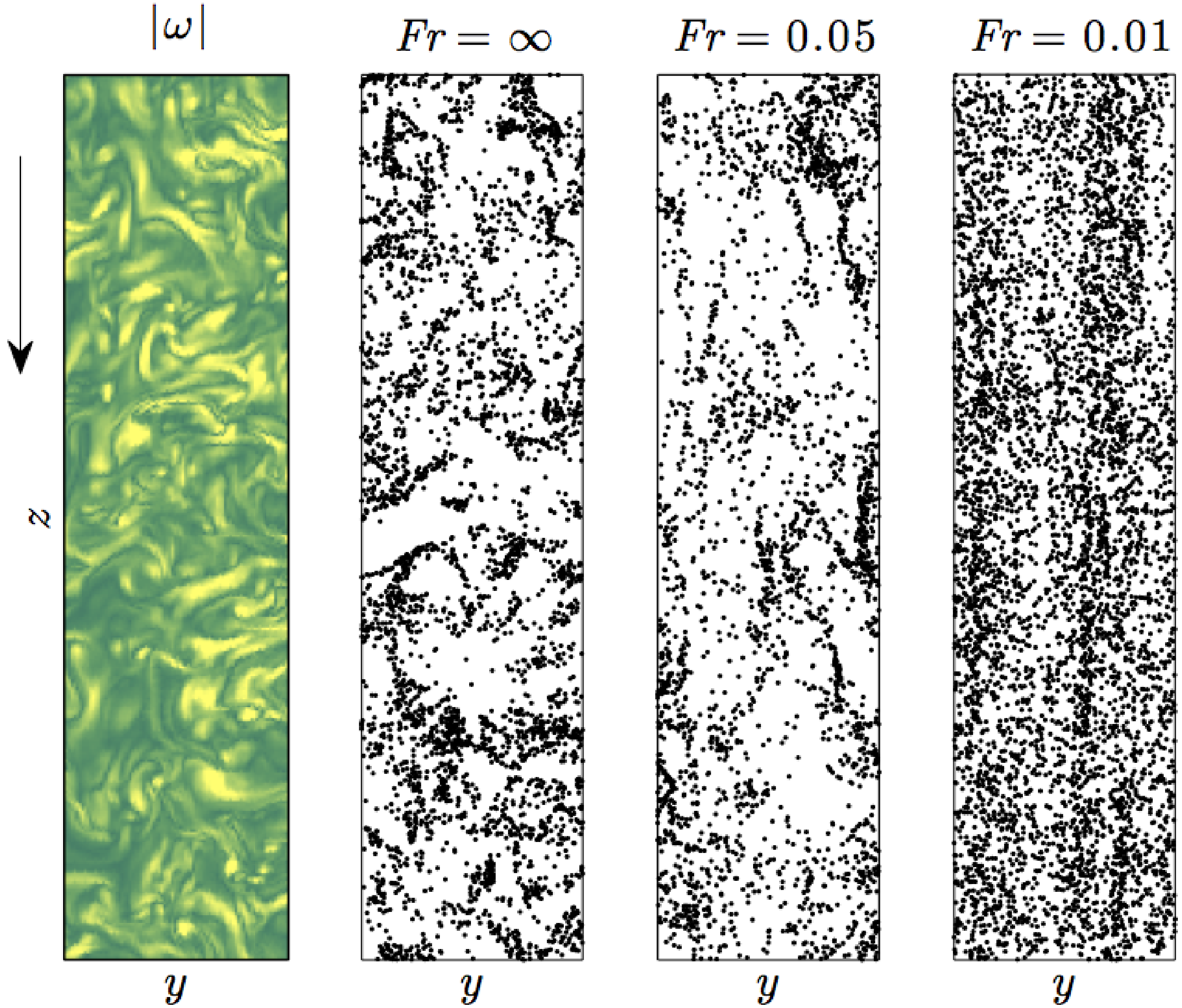
Simulation: Details

Re_λ	$u_{\rm rms}$	Δt	η	τ_η	L	T_L	N^3	N_p
460	0.189	0.0012	1.45×10^{-3}	0.083	1.85	9.9	2048^3	10×10^8
290	0.185	0.003	2.81×10^{-3}	0.131	1.85	9.9	1024^3	1.28×10^8
127	0.144	0.02	1.12×10^{-2}	0.45	2.11	14.6	256^3	0.08×10^8





Particle Distribution: Effect of Gravity



Snapshot of the vorticity modulus (Left; yellow = low values, green = high values) and of the particle positions for $R_\lambda = 130$, $\mathrm{St} = 1$ and three different values of the Froude number in a slice of thickness 10η, width 130η, and height 520η. The vertical arrow indicates gravity.

Bec, Homann, and Ray, Phys. Rev. Lett. **112**, 184501 (2014).

Settling Velocity: Qualitative Understanding

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- Define : The average settling velocity $V_g = -\langle \mathbf{V}_{\rm p} \cdot \hat{\mathbf{e}}_z \rangle$.
- Statistical stationarity $\implies$ $V_g = \tau_{\rm p} g - \langle u_z(\mathbf{X}_{\rm p}, t) \rangle$.
- Define : The relative increase in settling velocity:

$$\Delta_V = (V_g - \tau_{\rm p} g)/(\tau_{\rm p} g) = -\langle u_z(\mathbf{X}_{\rm p}, t) \rangle / (\tau_{\rm p} g)$$

- *If* settling particles in a turbulent flow sample regions where the vertical fluid velocity is aligned with gravity, we expect an enhancement of the average settling velocity.

M. Maxey, J. Fluid Mech. **174**, (1987).
L.-P. Wang & M. Maxey, J. Fluid Mech. **256**, 27 (1993).
K. Gustavsson, *et al.*, Phys. Rev. Lett. **112**, 214501 (2014).

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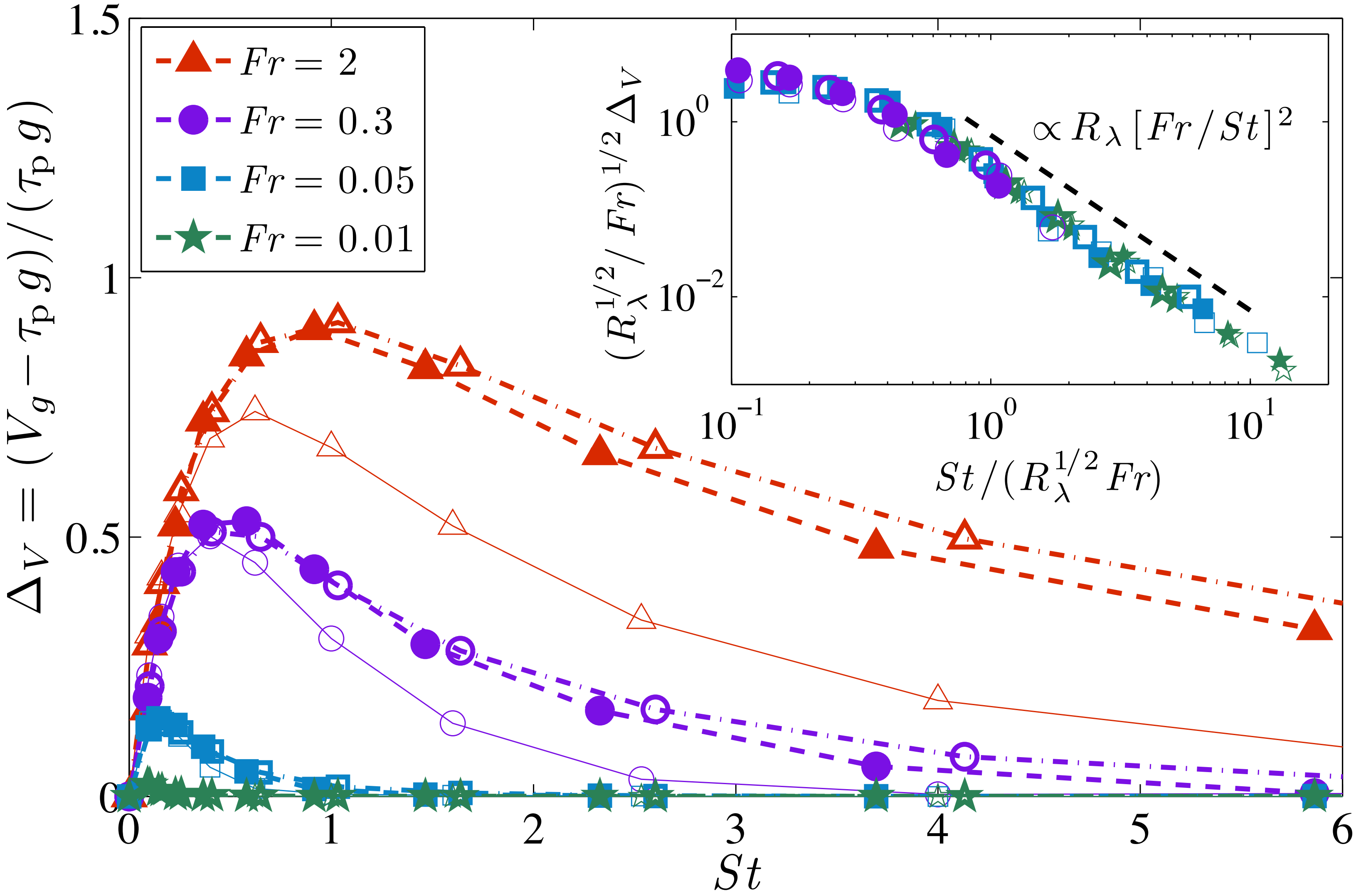
$$\Delta_V = (V_g - \tau_\mathrm{p} g)/(\tau_\mathrm{p} g) = -\langle u_z(\mathbf{X}_\mathrm{p}, t) \rangle / (\tau_\mathrm{p} g)$$

  - What is its dependence on the particle Stokes number and for different values of Fr and R_λ?

- *If* settling particles in a turbulent flow sample regions where the vertical fluid velocity is aligned with gravity, we expect an enhancement of the average settling velocity.
  - Is there a way to see this preferential sampling from the equations of motion?

M. Maxey, J. Fluid Mech. **174**, (1987).
L.-P. Wang & M. Maxey, J. Fluid Mech. **256**, 27 (1993).
K. Gustavsson, *et al.*, Phys. Rev. Lett. **112**, 214501 (2014).

Settling Velocity



Relative increase of the settling velocity Δ_V as a function of the Stokes number St for various Froude numbers, as labeled, and $R_\lambda = 130$ (thin symbols, plain lines), $R_\lambda = 290$ (filled symbols, dashed lines) and $R_\lambda = 460$ (open symbols, broken lines). Inset: $[R_\lambda^{1/2}/Fr]^{1/2}\Delta_V$ as a function of $St/[R_\lambda^{1/2}Fr]$ for the same data.

Bec, Homann, and Ray, Phys. Rev. Lett. **112**, 184501 (2014).

Settling Velocity: Preferential Sampling

Small Stokes Asymptotics

- **Why is there an enhancement?**
  - To leading order, the particles advected by an effective velocity field:

$$\mathbf{v} = \mathbf{u} + \tau_{\mathrm{p}}\mathbf{g} - \tau_{\mathrm{p}}\left[\partial_t \mathbf{u} + (\mathbf{u} + \tau_{\mathrm{p}}\,\mathbf{g}) \cdot \nabla \mathbf{u}\right].$$

  - Focus on the (x, y) plane.
  - By using isotropy and incompressibility, we obtain:

$$\langle u_z \nabla_\perp \cdot \mathbf{v}_\perp \rangle = \tau_{\mathrm{p}}^2 g \left\langle (\partial_z u_z)^2 \right\rangle > 0.$$

Settling Velocity: Preferential Sampling

Small Stokes Asymptotics

- **Why is there an enhancement?**
  - To leading order, the particles advected by an effective compressible velocity field:

$$\mathbf{v} = \mathbf{u} + \tau_\mathrm{p}\mathbf{g} - \tau_\mathrm{p}\left[\partial_t \mathbf{u} + (\mathbf{u} + \tau_\mathrm{p}\,\mathbf{g}) \cdot \nabla \mathbf{u}\right].$$

  - Focus on the (x, y) plane.
  - By using isotropy and incompressibility, we obtain:

$$\langle u_z \nabla_\perp \cdot \mathbf{v}_\perp \rangle = \tau_\mathrm{p}^2 g \left\langle (\partial_z u_z)^2 \right\rangle > 0.$$

  - *Particles preferentially cluster (negative divergence), on average, in the (x, y) plane, at points where the fluid velocity is vertically downwards ($u_z < 0$).*

L.-P. Wang & M. Maxey, J. Fluid Mech. **256**, 27 (1993).
K. Gustavsson, *et al.*, Phys. Rev. Lett. **112**, 214501 (2014).

Settling Velocity: Quantitative Understanding

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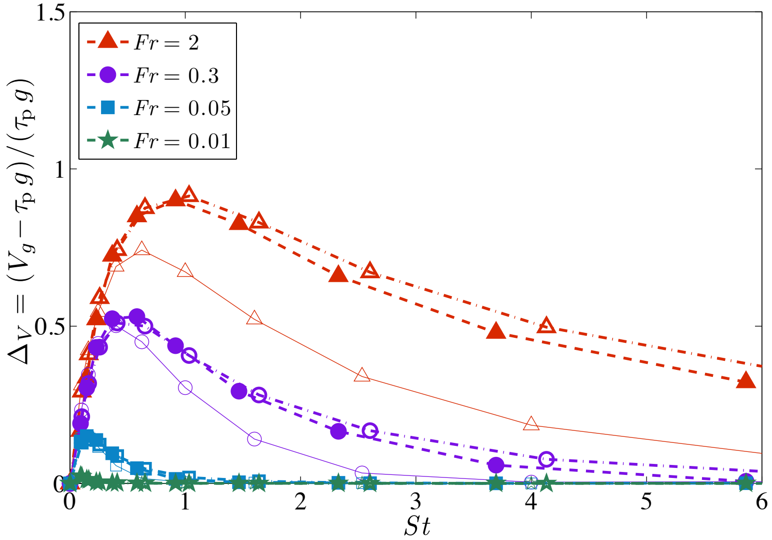

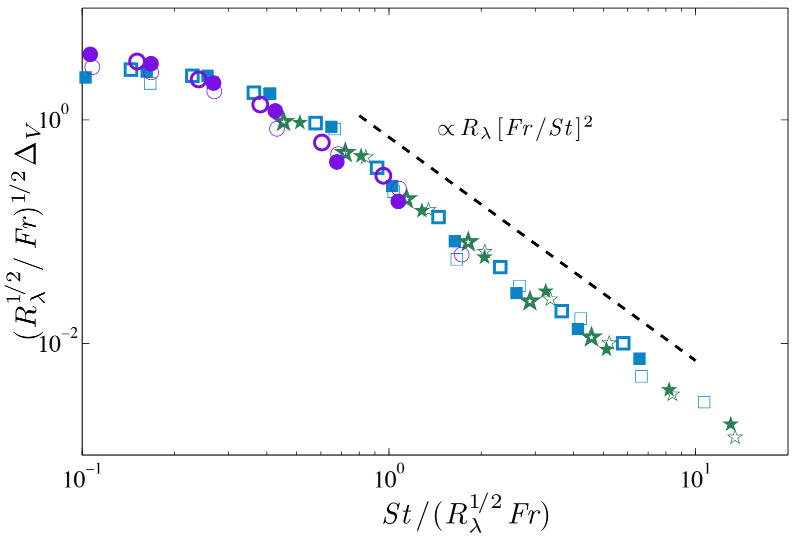

Small Stokes Asymptotics

$$\boxed{\Delta_V \propto \tau_\eta \tau_p \left\langle (\partial_z u_z)^2 \right\rangle \propto St}$$

Assumptions & Algorithm:

- Relate V_g to $\langle u_z \nabla_\perp \cdot \mathbf{v}_\perp \rangle$.
- Hence $\langle u_z(\mathbf{X}_p, t) \rangle \propto \tau_\eta \langle u_z \nabla_\perp \cdot \mathbf{v}_\perp \rangle$.

G. Falkovich, *et al*, Nature **419**, (2002).

Large Stokes Asymptotics

$$\boxed{\Delta_V \propto R_\lambda^{3/4} Fr^{5/2} St^{-2}}$$

Valid:

- $St \gg R_\lambda^{1/2} Fr$ and $Fr \ll R_\lambda^{1/2}$.

I. Fouxon & P. Horvai, Phys. Rev. Lett. **100**, (2008).

Small-scale, Two-particle Statistics

- Describe the evolution of pair separations in terms of $\nabla \mathbf{u}$.
- $V_g \gg u_\eta$: the particles travel η in a time shorter than τ_η.
- Rescale time by $\tau_\eta(V_g/u_\eta)$ and space by η:

$$\frac{\mathrm{d}^2\mathbf{R}}{\mathrm{d}s^2} \simeq -\frac{1}{\tilde{S}}\left[\frac{\mathrm{d}\mathbf{R}}{\mathrm{d}s} - \mathbf{R}\cdot\boldsymbol{\sigma}(s)\right],$$

  where $\boldsymbol{\sigma}$ is a Gaussian tensorial noise with co-variance $\langle \sigma_{ij}(s)\sigma_{k\ell}(s')\rangle = (\nu/\varepsilon)\langle \partial_i u_j \partial_k u_\ell\rangle \delta(s-s')$.
- The effective Stokes number $\tilde{S} = St\,(u_\eta/V_g)$.
- $V_g \gg u_\eta$: small-scale two-particle statistics depend only on $\tilde{S}$.
- When $\Delta_V \ll 1$, $\tilde{S} \simeq Fr$; the statistics become independent of St when $St \gg Fr$.

Observable

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$\mathcal{D}_2$: The Correlation Dimension

- An important observable measuring particle clustering is the correlation dimension $\mathcal{D}_2$ of their spatial distribution.
- It is given by $\mathbb{P}_2(r) \propto r^{\mathcal{D}_2}$ for $r \ll \eta$, where $\mathbb{P}_2(r)$ is the probability that two particles are within a distance r.

K. Gustavsson, *et al.*, Phys. Rev. Lett. **112**, 214501 (2014).

$\mathcal{D}_2$: Correlation Dimension

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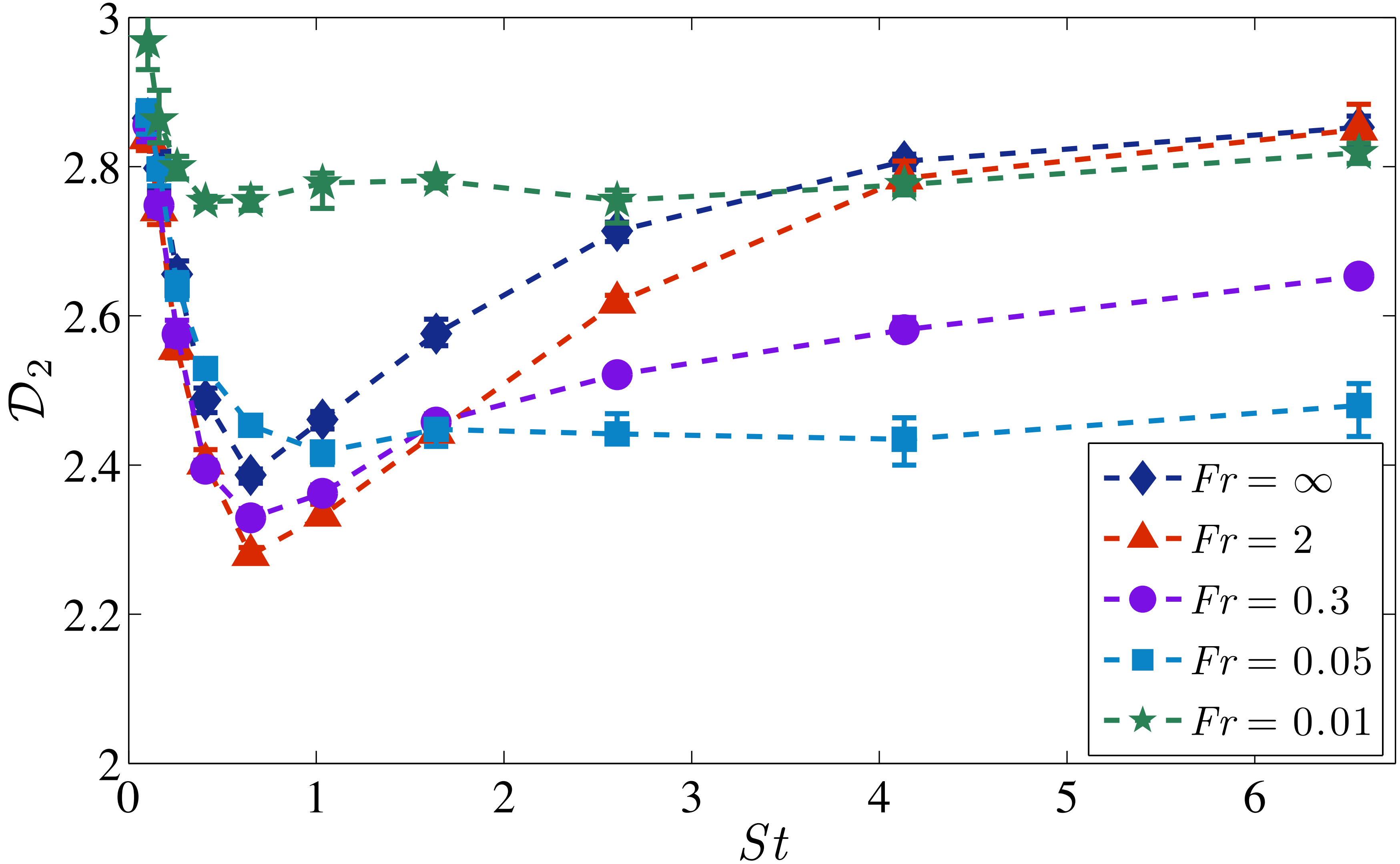

Correlation dimension $\mathcal{D}_2$ of the particle distribution as a function of the Stokes number for $R_\lambda = 460$ and various Froude numbers as labeled. Smaller Reynolds numbers (not shown here) display a similar behavior.

Bec, Homann, and Ray, Phys. Rev. Lett. **112**, 184501 (2014).

$\mathcal{D}_2$: What it tells us

- Gravity acts in a non-uniform manner.
- It tends to enhance concentration (decrease $\mathcal{D}_2$) when both the Stokes and the Froude numbers have moderate values.
- When $Fr \ll 1$, clustering is decreased for $St \lesssim 1$ and increased for $St \gtrsim 1$.
- For all finite Fr, one observes that $\mathcal{D}_2$ saturates to a finite value when $St \to \infty$.
- For $V_g \gg u_\eta$, the fractal dimension $\mathcal{D}_2$ is a function of the effective Stokes number $\tilde{S}$ only, which for $St \gg Fr$ becomes independent of St.
- In this asymptotics, the correlation dimension depends solely on Fr.
- The limiting value of $\mathcal{D}_2$ is a non-monotonic function of Fr.

Implication

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- The increase in clustering observed for order-unity values of *St* and *Fr* means that settling can significantly impact the timescales of interaction between particles.
- When interested for instance in the collisions, estimations of the geometrical rate involve the probability density that two particles are at a distance $r = 2a$ equal to the sum of their radii and thus scales as $(2a)^{\mathcal{D}_2 - 1}$.
- However, this quantity alone is not enough as the collision rate involves also the typical velocity at which particles approach each other.
- Indeed, for same-size particles, it is given by setting $r = 2a$ in the approaching rate.

$$\kappa(r) = -\left\langle w\,\theta(-w)\,\delta(|\mathbf{R}| - r)\right\rangle ,$$

where $w = \mathrm{d}|\mathbf{R}|/\mathrm{d}t$ is the longitudinal velocity difference between particles, θ the Heaviside function, and $\langle\cdot\rangle$ the average over all particle separations $\mathbf{R}$.

Observable

The Approaching Rate

- $\kappa(r) = -\langle w\,\theta(-w) \,|\, |\mathbf{R}| = r\rangle\,(\mathrm{d}\mathbb{P}_2/\mathrm{d}r)$, where $w = \mathrm{d}|\mathbf{R}|/\mathrm{d}t$ is the longitudinal velocity difference between particles, θ the Heaviside function, and $\langle\cdot\rangle$ the average over all particle separations $\mathbf{R}$
- This last quantity behaves also as a power of r for $r \ll \eta$ with an exponent ξ_1 given by the first-order structure function of particle velocities.
- This implies that $\kappa(r) \sim r^{\gamma}$ with $\gamma = \xi_1 + \mathcal{D}_2 - 1$.
- The dependence of γ upon St, which encompasses particle clustering and velocity differences statistics, determines how the collision rate depends on the particles size and inertia.

K. Gustavsson, *et al.*, Phys. Rev. Lett. **112**, 214501 (2014).

Approaching Rates

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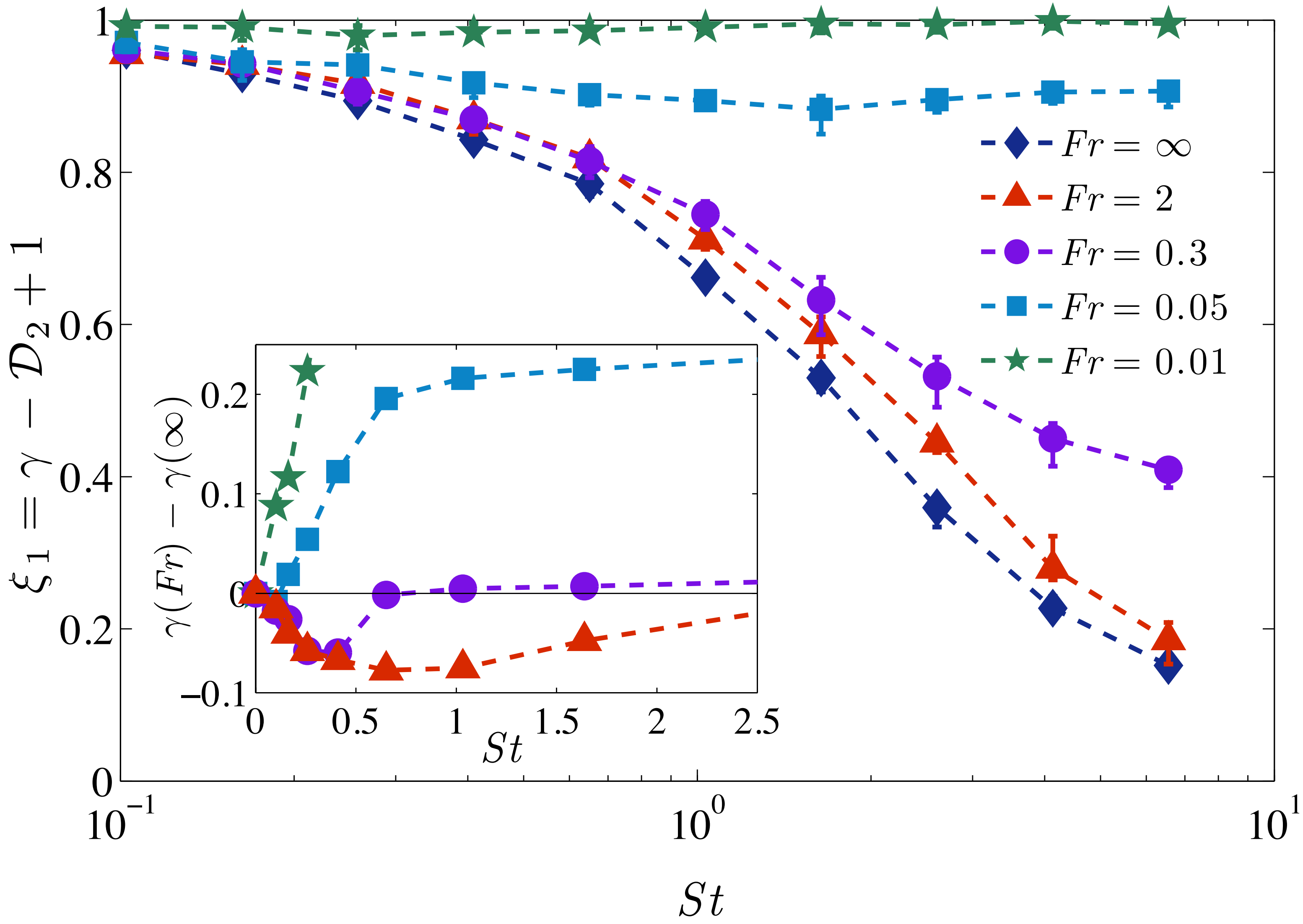


Exponent of the velocity difference $\xi_1 = \gamma - \mathcal{D}_2 + 1$ as a function of the Stokes number for different *Fr* and $R_\lambda = 460$. Inset: difference between the approaching rate exponent γ associated to the different values of *Fr* and that associated to particles feeling no gravity ($Fr = \infty$).

Bec, Homann, and Ray, Phys. Rev. Lett. **112**, 184501 (2014).

Understanding Approaching Rates

- $\kappa(r) \sim r^{\gamma}$ with $\gamma = \xi_1 + \mathcal{D}_2 - 1$.
- For $Fr = \infty$, $\xi_1 = 1$ at small St (tracers) and $\xi_1 = 0$ for $St \to \infty$ (scale-independent velocity differences).
- When Fr decreases, the effective Stokes number decreases, so that particles get closer to tracers of the effective flow and $\xi_1 \to 1$.

M. Wilkinson, *et al.*, Phys. Rev. Lett. **97**, 048501 (2006).
J. Bec, *et al*, J. Fluid Mech. **646**, 527 (2010).
J. Bec, *et al*, Phys. Fluids **17**, 073301 (2005).

Approaching Rates: Competing Mechanisms

- The two mechanisms determining the rate at which particles collide, namely preferential concentration and large velocity differences, are thus affected in a competing manner by gravity.
- The enhancement of particle clustering takes over the decrease of velocity differences when $St \lesssim Fr$.
- Hence, $\gamma(Fr) < \gamma(\infty)$ for $St \lesssim Fr$, indicating that the collision rates between same-size particles are larger in the presence of gravity.
- These corrections are responsible for an important increase of the geometrical collision rate.
  - Example: In the settings of a highly-turbulent cloud, namely $Fr = 0.3$ and $St = 0.4$, the collision rate doubles when the effect of gravity is included.

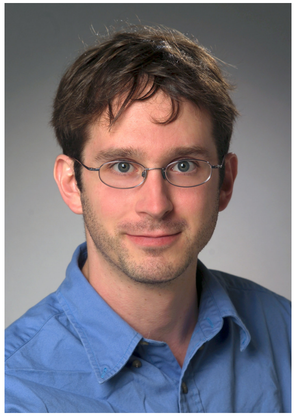
Conclusions

- Heavy particles suspended in a turbulent flow settle faster than in a still fluid.
- This effect stems from a preferential sampling of the regions where the fluid flows downward and is quantified as a function of the level of turbulence, of particle inertia, and of the ratio between gravity and turbulent accelerations.
- By using analytical methods and detailed numerical simulations, settling is shown to induce an effective horizontal two-dimensional dynamics that increases clustering and reduce relative velocities between particles.
- These two competing effects can either increase or decrease the geometrical collision rates between same-size particles and are crucial for realistic modeling of coalescing particles.

Open Questions

- The functional form of the velocity difference PDF depends on the value of the Stokes number:
  - Extending arguments for synthetic flows and in the large *St* limit to moderate values of the Stokes numbers in real flows for which the contribution of inertial-range and dissipative-scale statistics cannot be neglected.
- The role of intermittency of turbulent velocity statistics and non-trivial Reynolds number dependencies of particle relative velocity and collision statistics.
- Coalescences.
- Modelling collision kernels.

Acknowledgements



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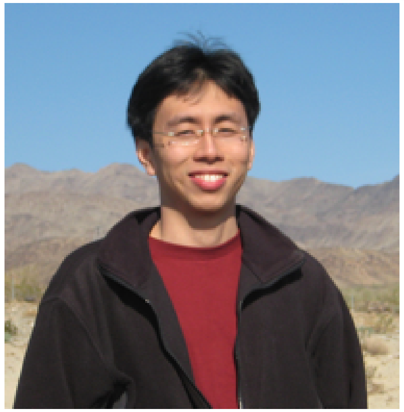

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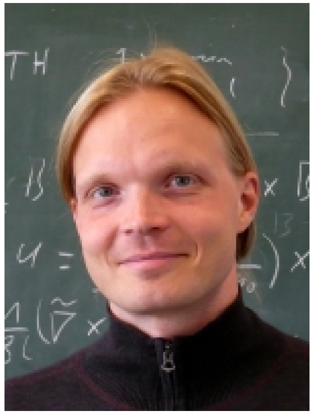

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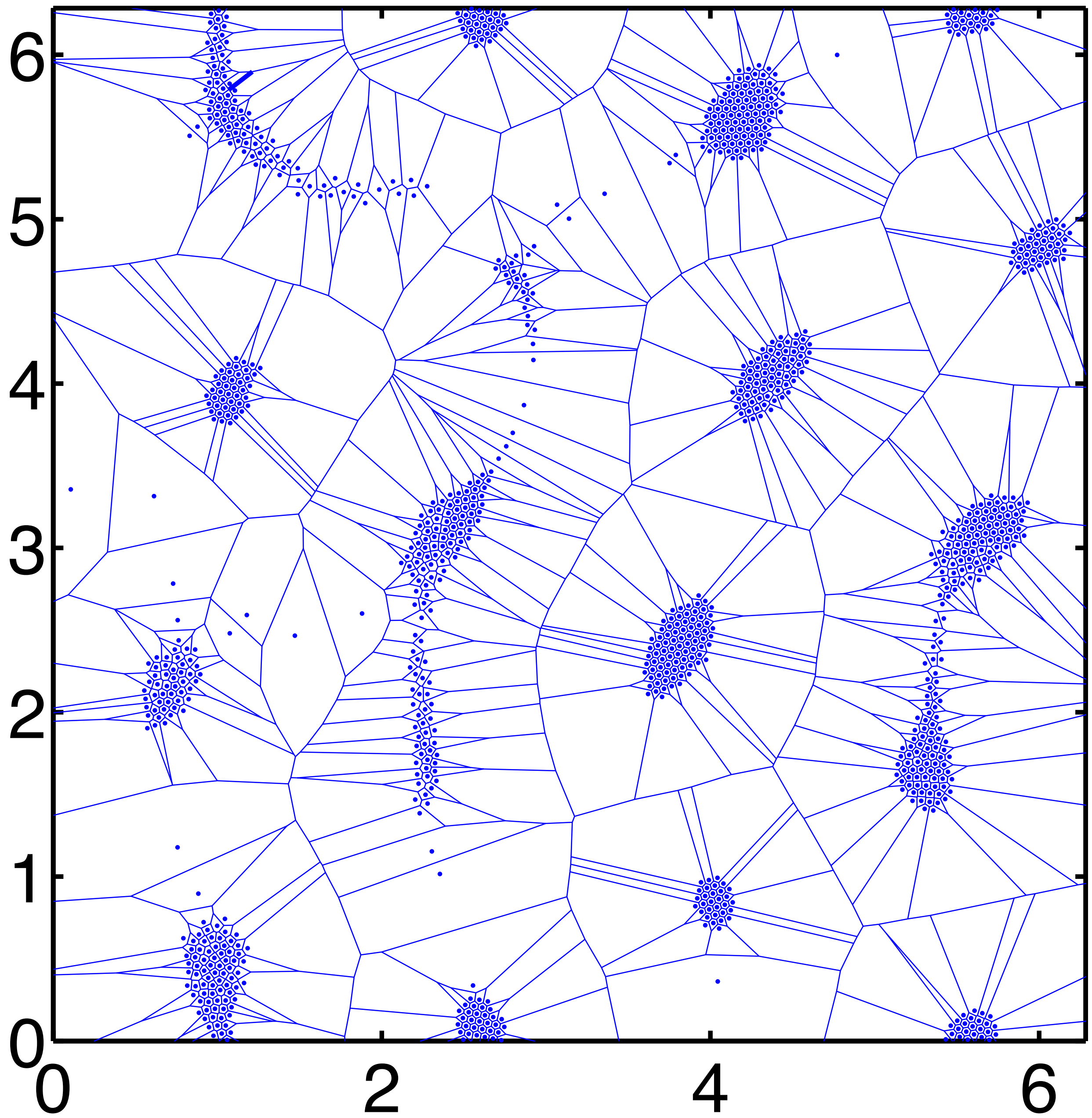

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