PROBABILITY DENSITY FUNCTION FOR PARTICLE ACCELERATIONS IN TURBULENCE AND THE STRUCTURE OF VORTICES

Gerardo Ruiz Chavarria

Departamento de Física

Facultad de Ciencias, UNAM

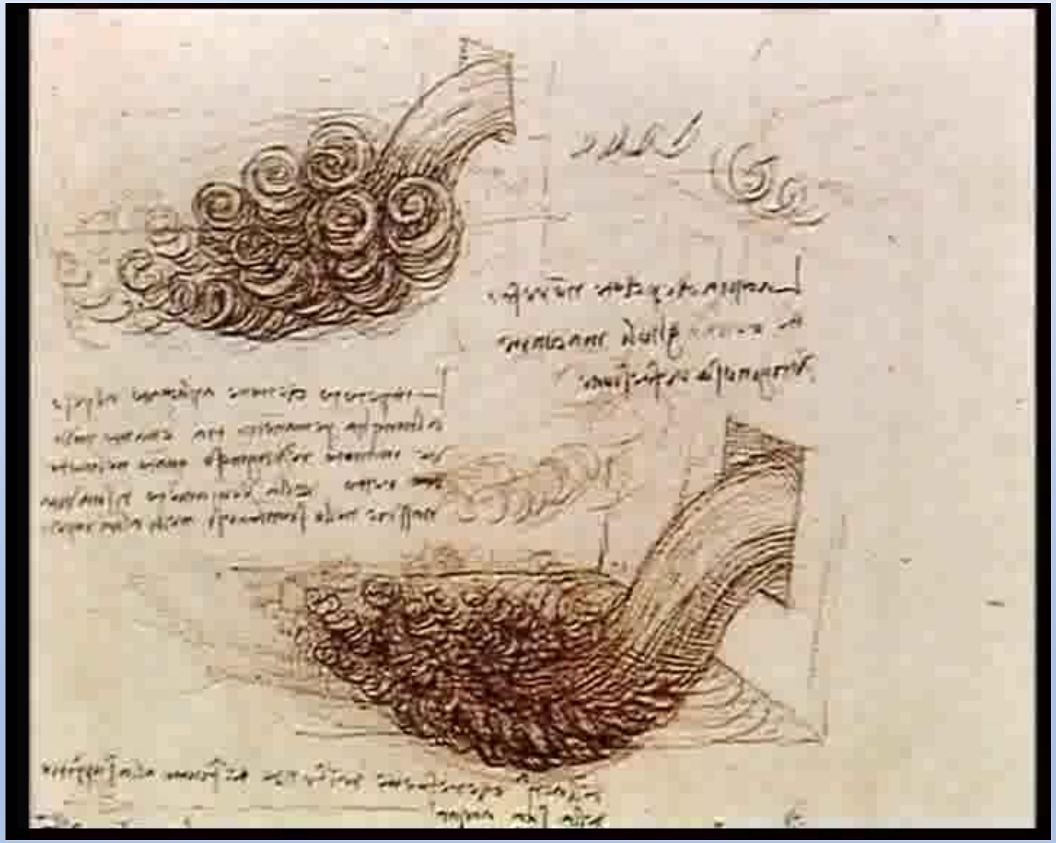
MEXICO

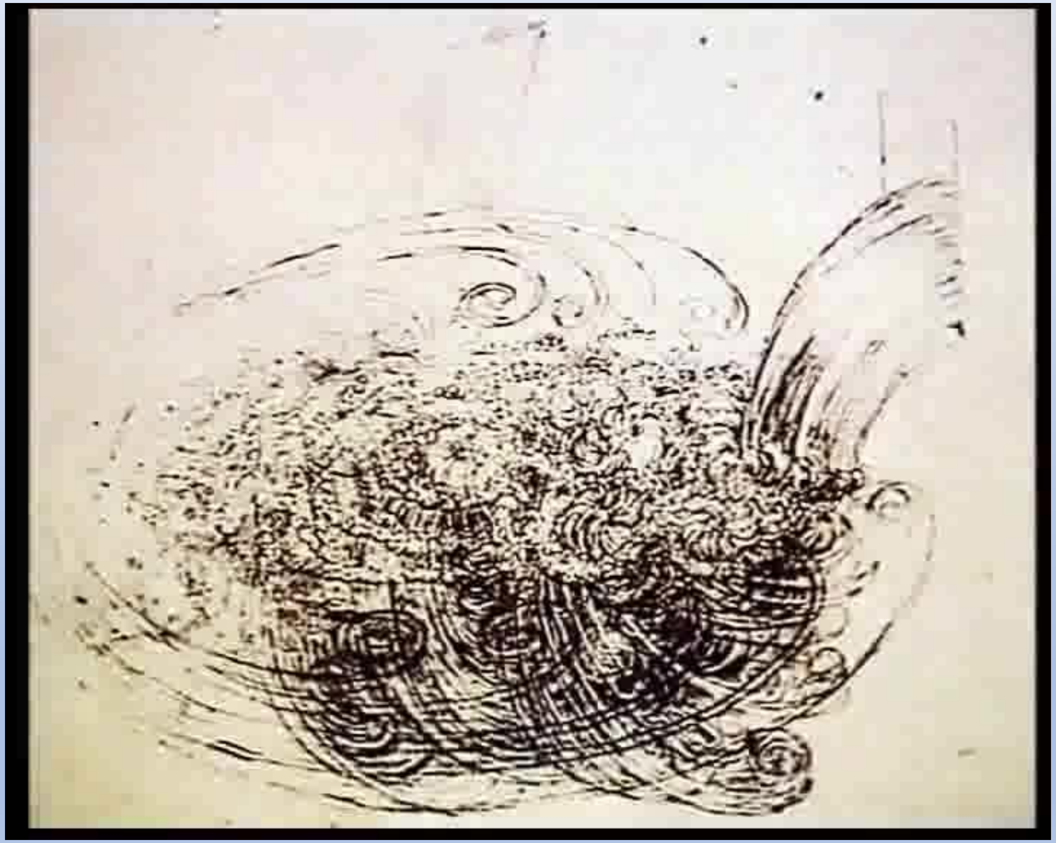
e-mail: gruiz@unam.mx

ABSTRACT

In the last decades many investigations about the motion of particles in a turbulent flow have been conducted. The use of high speed cameras and Doppler sonar have allowed to measure the velocity and the acceleration of particles inside the fluid. From the experimental data, the probability density function (pdf) of the particle acceleration has been computed. The pdf is strongly non gaussian, with accelerations as high as 200g . The existence of these strong accelerations has been related to the trapping of particles into the vortex cores. To support this assertion we need to recover a classic picture of a vortex. In a first approximation the particles inside the vortex core (after subtracting its translation velocity) move as in a rigid body rotation, the fluid particles move along circular paths. In this study we have investigated the structure of annular vortices for different values of the Reynolds number (Re), but always in a laminar regime. The vortices are produced by expelling the fluid contained in a box through a circular hole. The velocity has been measured using both Particle Image Velocimetry (PIV) and Hot Wire Anemometry, the latter having a better spatial resolution (we have used a probe with a diameter of 5 mm and 0.5 mm long). In the vortex core the tangential velocity is proportional to the distance to its center. For an experiment corresponding to a Reynolds number Re=1200 we have observed that the tangential velocity changes from -1 m/s to 3 m/s through a distance of 4 mm. In this case, the acceleration of fluid particles (due to the centrifugal effect) in the outer limit of the core attains the value $a = 2^2/2^{-3} = 2000\ m/s^2$. We then investigate the size of the vortex core and the acceleration of fluid particles (for vortices having different values of Reynolds number. We discuss the trend observed when the Reynolds number increases and the role of the vortex stretching to produce strong accelerations in a turbulent flow.

The word turbulence (LA TURBULENZA) appears in the works of Leonardo da Vinci:





A well known phrase by Lewis Richardson

Big whirls have little whirls
that feed on their velocity
Little whirls have lesser whirls
and so on to viscosity
-**In the molecular sense**

(Richardson L.F. Weather prediction by numerical process Ph D Thesis. Cambridge University Press, 1922)

Some results in turbulence

- Eulerian framework

  Scaling of structure function in the inertial range:

$$\langle \delta V(r)^p \rangle \sim r^{\varsigma_p}$$

  Spectra of the kinetic energy

$$F(k) \sim k^{-5/3}$$

Probabilty density function of velocity differences $\boldsymbol{\delta V(r)}$

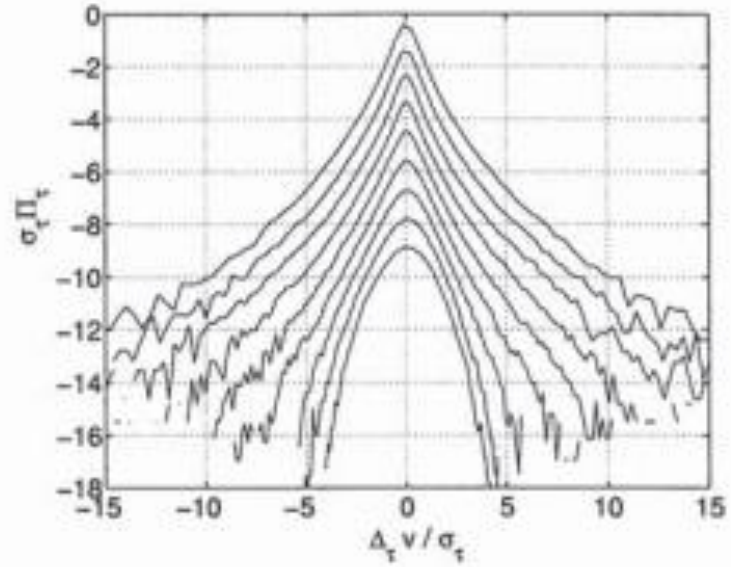
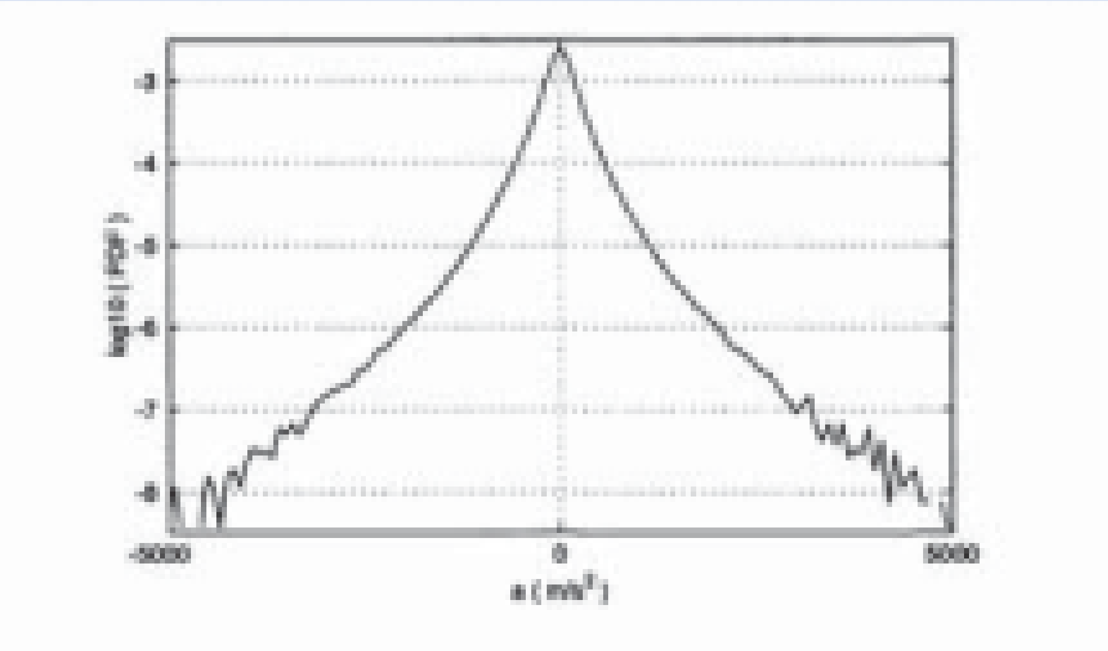


FIGURE 8. PDF $\sigma_\tau \Pi_\tau$ of the normalized increment $\Delta v_\tau / \sigma_\tau$. The curves are shifted for clarity. From top to bottom: $\tau = [0.15, 0.3, 0.6, 1.2, 2.5, 5, 10, 20, 40]$ ms.

1) N. Mordant, P. Metz, J.F Pinton & O Michel, Lagrangian measurement in fully developed turbulence. AIP Conference Proceedings, 622, 343, (2002).

- Lagrangian framework

  Probability density function af particle accelerations:



- According to dimensional arguments the scaling for particle accelerations is:

$$a_r \sim \epsilon^{2/3} r^{-1/3}$$

For the experiments of the previous figure the prediction for acceleration at the Kolmogorov scale is $\sim 300 \frac{m}{s^2}$. However, the measurement carried out with an ultrasonic sonar gives accelerations as high as $2000 \frac{m}{s^2}$

- Scaling for lagrangian structure functions:

$$< \delta V(\tau)^p > \sim \epsilon^{p/2} \tau^{p/2}$$

- **IT IS POSSIBLE TO RECOVER STRONG ACCELERATIONS IN A LAMINAR FLOW?**

We investigate two cases

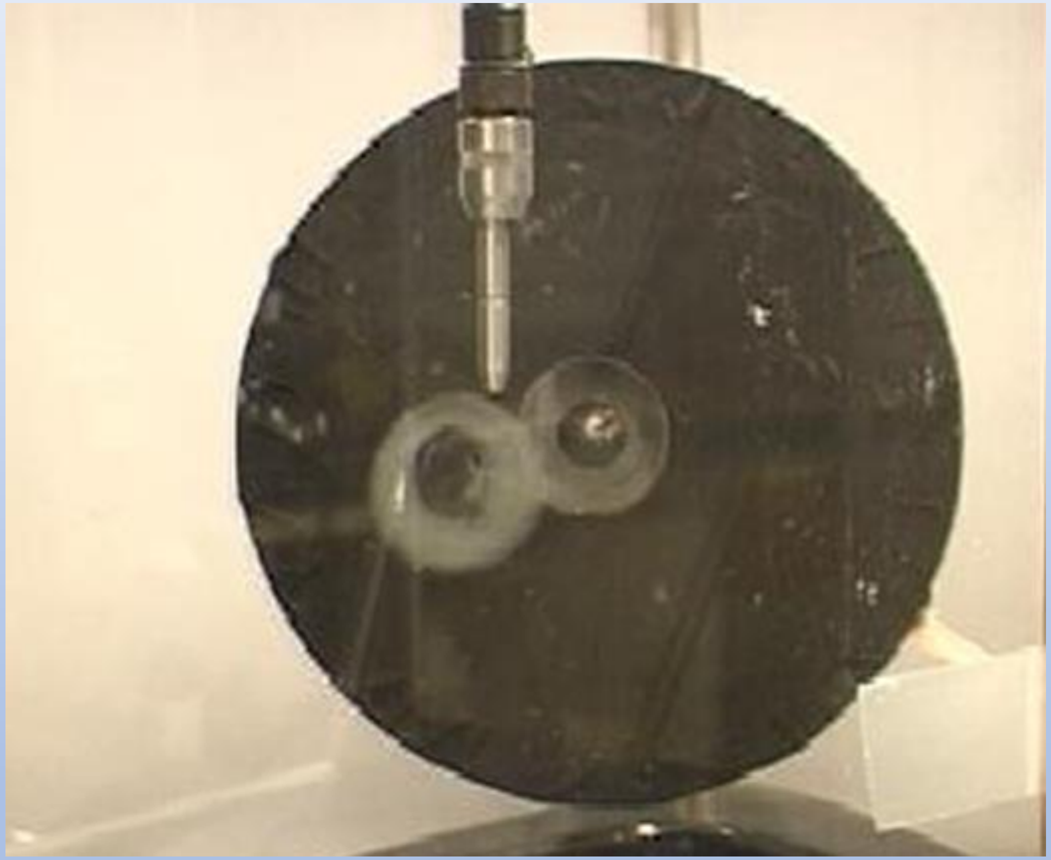
- Vortices at the outlet of a channel
- Annular vortices

- The answer is YES

- In this talk we present result for annular vortices

EXPERIMENTAL SETUP

- The annular vortices are produced by expelling the air contained in a cavity. The cavity is built with an electronic speaker in front of which there is a wall with a circular hole of diameter 15 mm. The fluid is expelled from the box when an electric pulse is applied to the speaker. Then, a toroidal vortex emerges from the cavity with a speed between 0.8 and 2.0 m/s, depending of the amplitude of the pulse. To investigate the properties of the annular vortices we use both Particle Image Velocimetry (PIV) and Hot Wire Anemometry (HWA).
- The Reynolds numbers covered in our experiments are in the range from 800 to 2200 (laminar regime)



Methods to mesure the velocity

- Particle Image Velocimetry (PIV)

  Particles used in water: glass hollow spheres (10, 20,50 μm)

  Particles used in air: titanium dioxide particles, olive oil drops

To iluminate a plane in the flow we use a laser with a line emisión at 520 nm. The power of laser is 4 W

We use a high speed camera (1500 fps and a resolution of 800 x600 pixels)

- HOT WIRE ANEMOMETRY

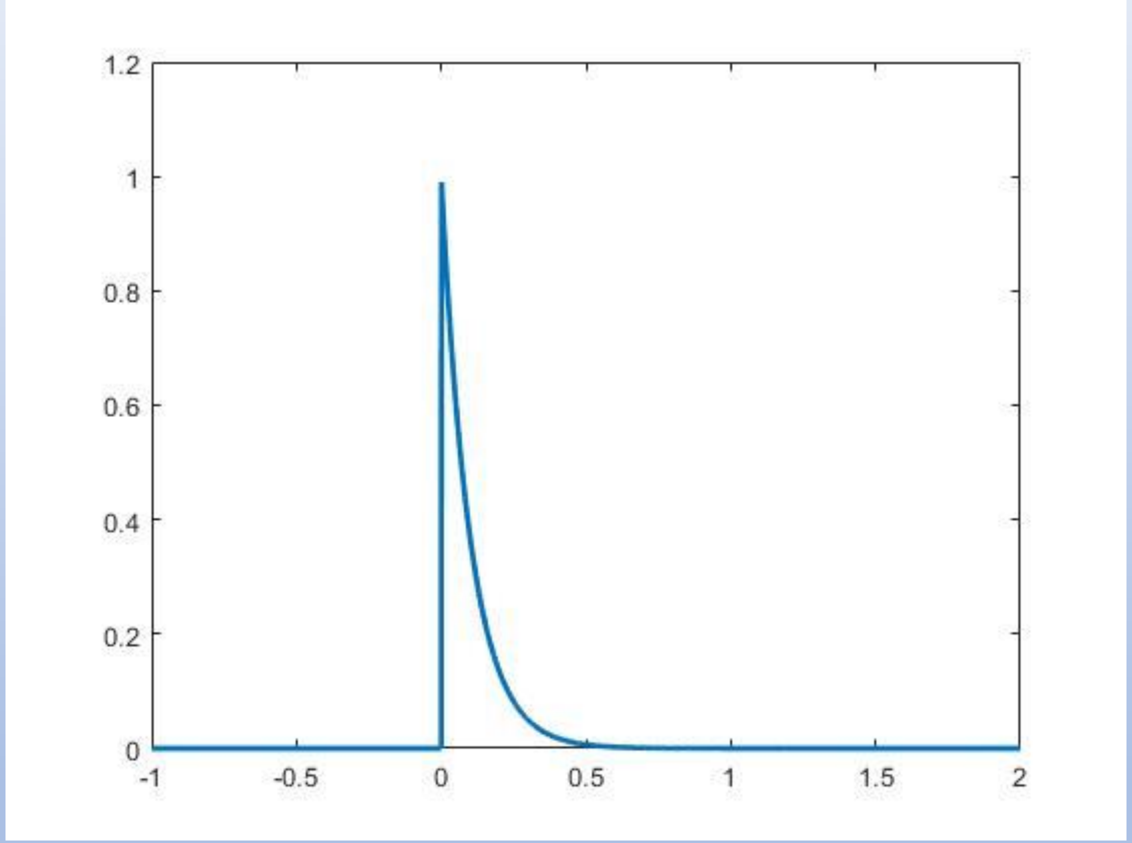
Size of the sensor: 0.5 mm long, diameter 5 µm

The hot wire anemometry allows to measure 1, 2 or 3 components of velocity in a point

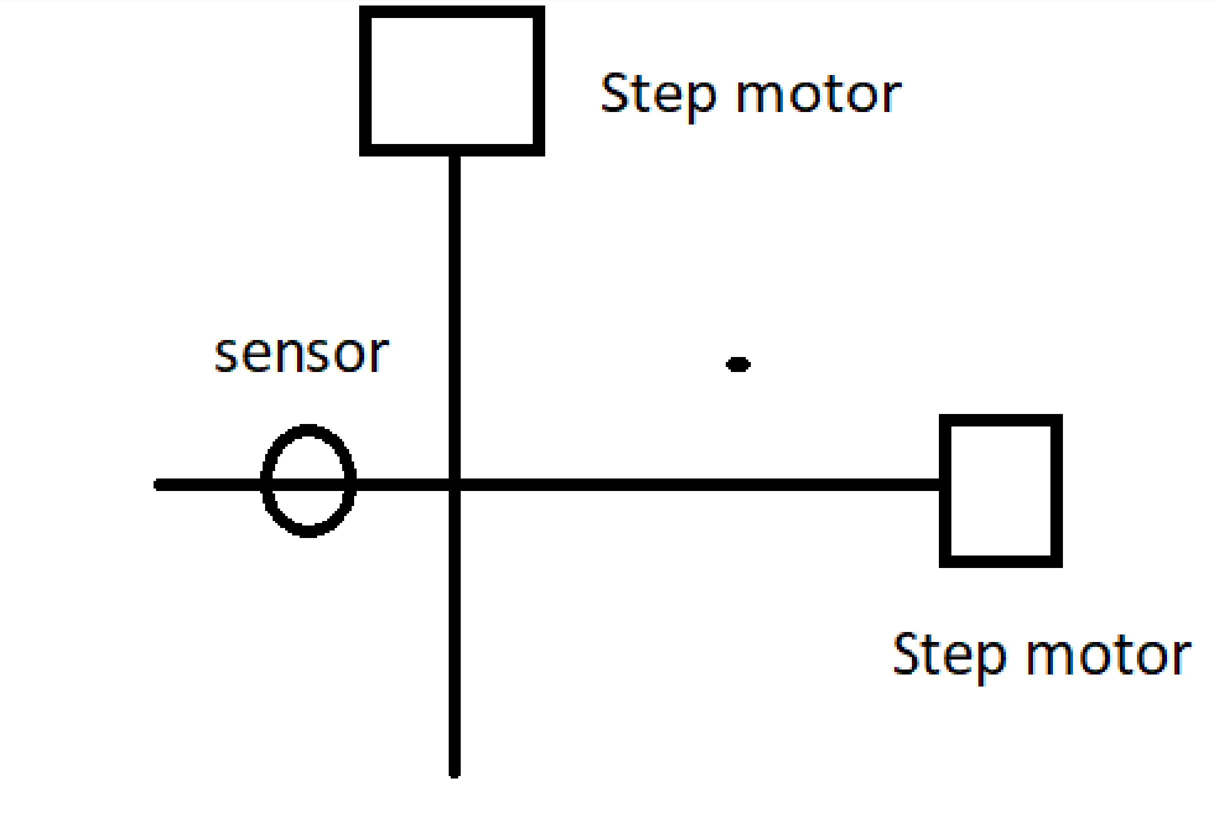
In a phenomenon which exhibits reproducibility , it is possible to measure the velocity with a hot wire probe in different points. In our case the data acquisition is synchronized with the perturbation applied to the cavity.

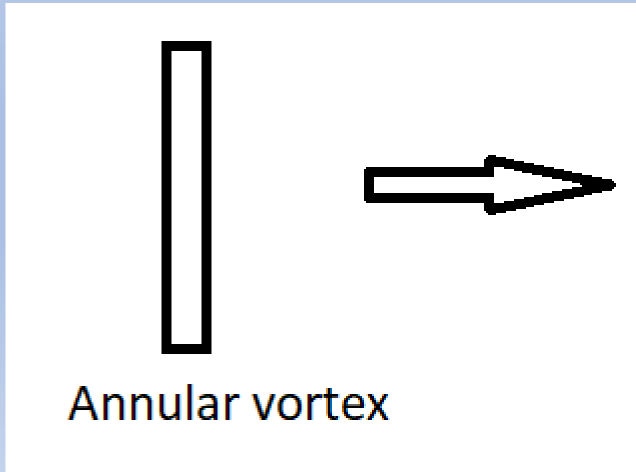
To measure the velocity in different points of a plane we use a x-y positioning system. The position of the sensor is controlled with two step motor. The resolution of the position is 0.1 mm

- Shape of the signal sent to the spéaker for the production of the annular vortex:



$$V(t) = 0 \;\; for\, t < 0 \qquad V(t) = V_0 \exp(-kt) \;\; for\, t > 0$$

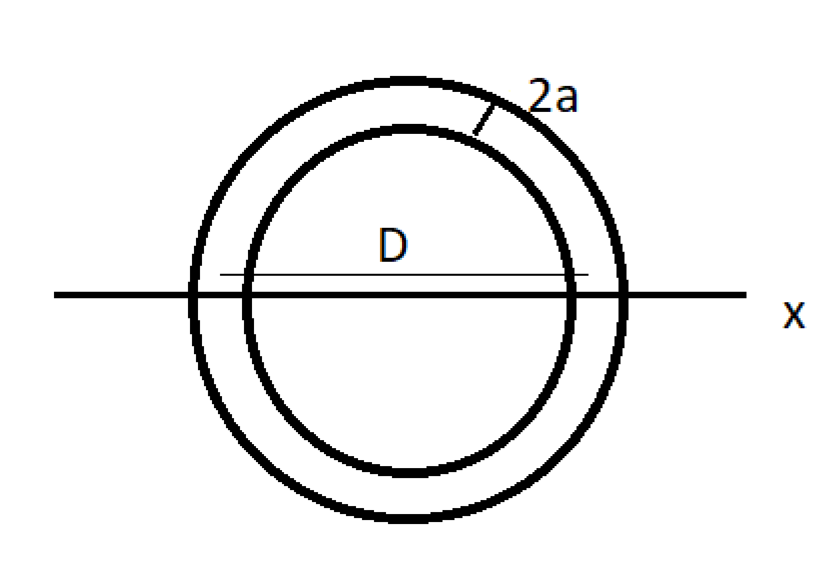




Velocity data are acquired at a rate of 4 kHz during 1.5 s

The data acquisition begins when the pulse is sent to the electronic speaker

We measure a component of the velocity (the axial vellocity)



a is the diameter of the vortex core
D is the diameter of the annular vortex

Reynolds number $Re = \frac{U\,D}{\nu}$

where U is the speed of the vortex and ν is the kinematic viscosity of the fluid

To evaluate the acceleration of fluid particles we measure the axial velocity on a straight line passing through the center of the annular vortex. Inside the core the axial velocity has a linear dependence on the x coordinate.

ANALOGY WITH THE ELECTROMAGNETIC THEORY

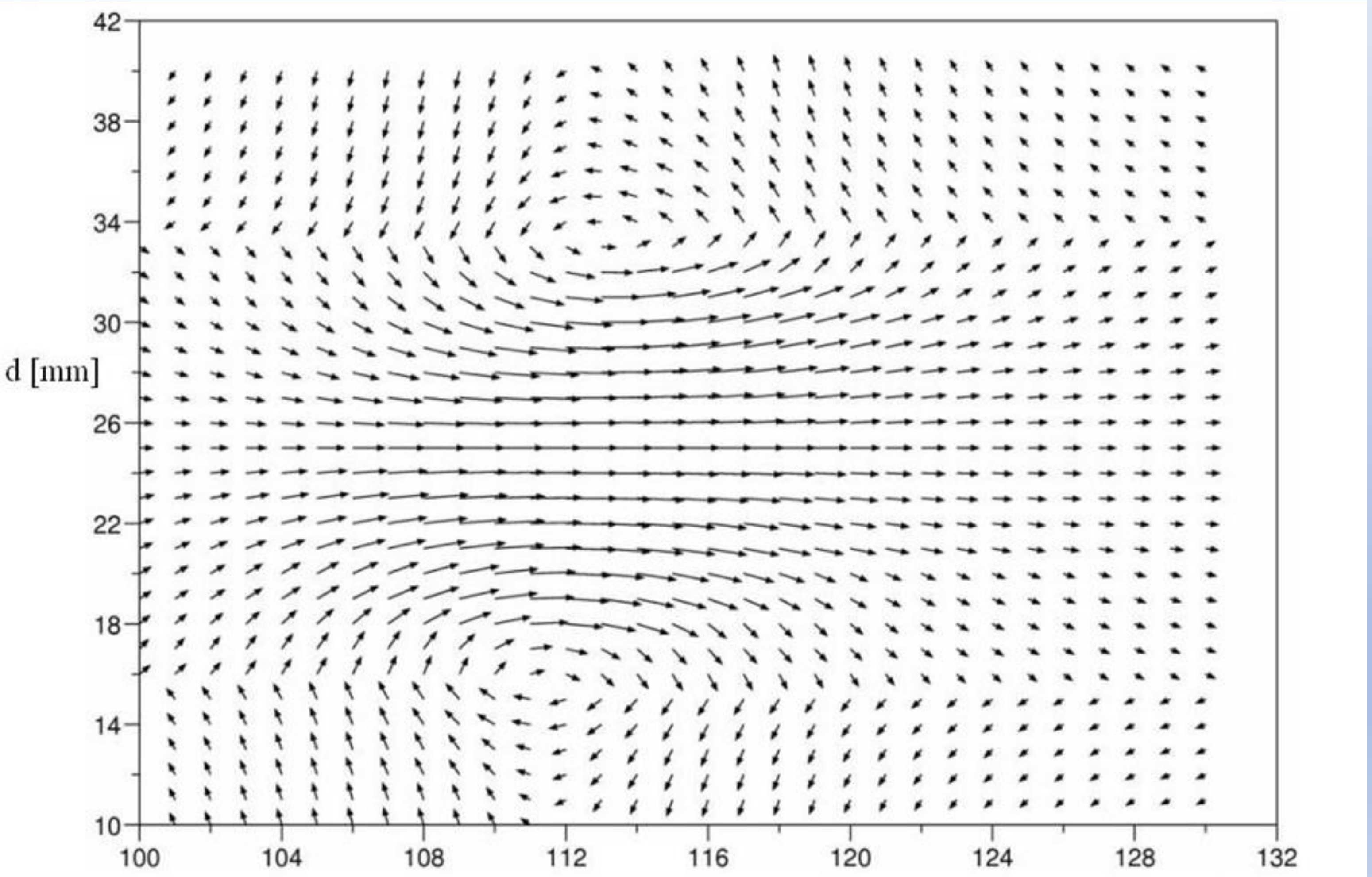
- $div\ \bar{B} = 0$ $\qquad$ $div\ \bar{u} = 0$
- $curl\ \bar{B} = \mu_0 \bar{J}$ $\qquad$ $curl\ \bar{u} = \bar{\omega}$

Biot-Savart law for fluids:

$$\bar{u} = \frac{\Gamma}{4\pi} \int \frac{d\bar{l} \times \left(\bar{r} - \bar{r'}\right)}{\| \bar{r} - \bar{r'} \|^{3/2}}$$

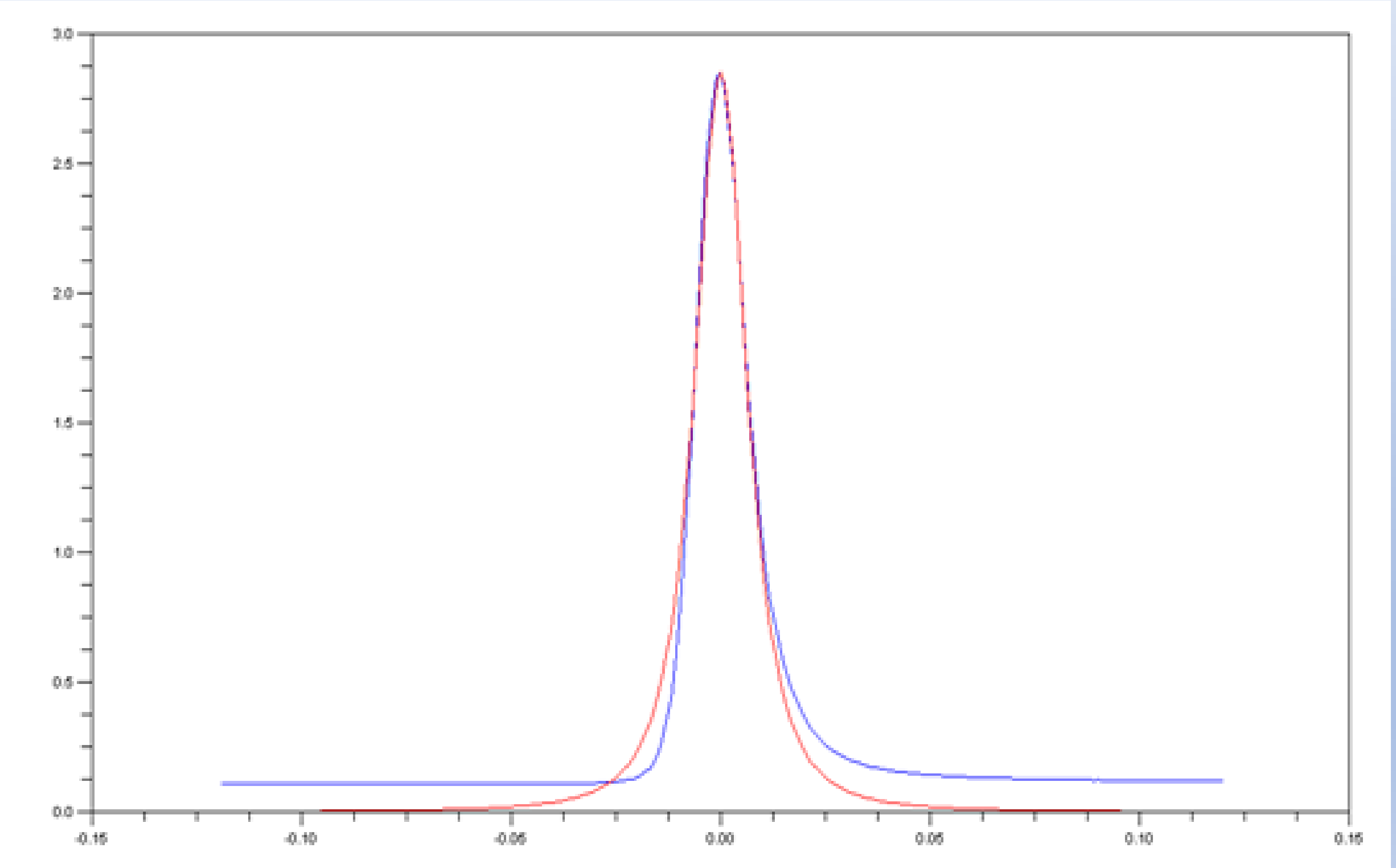
RESULTS

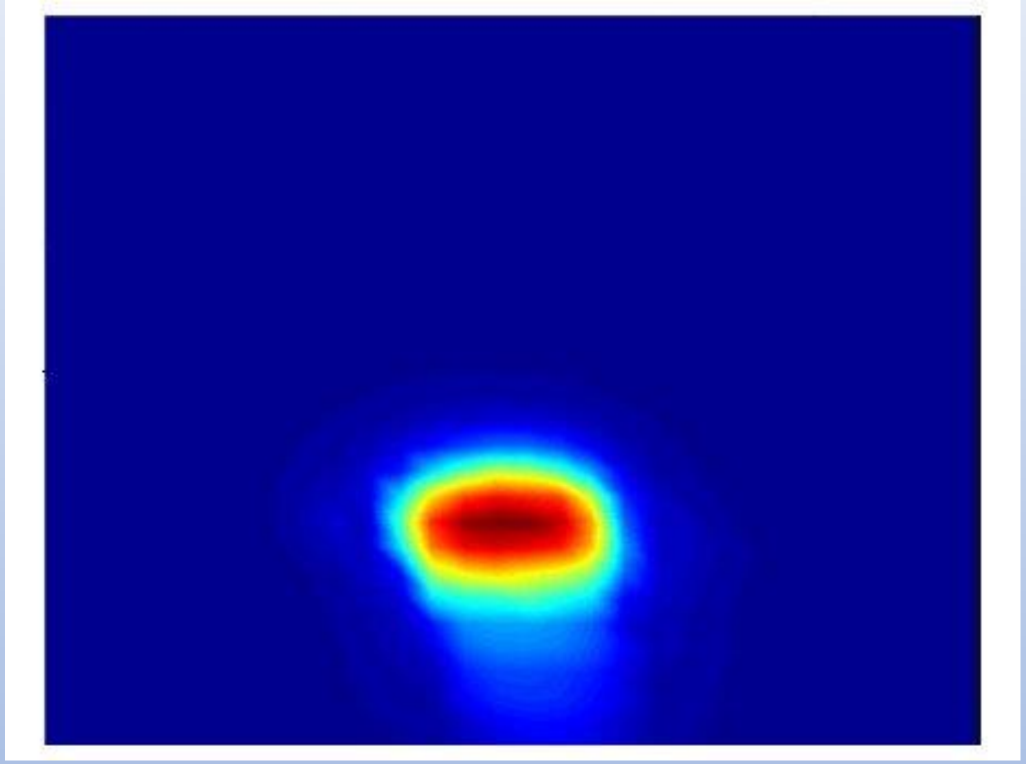
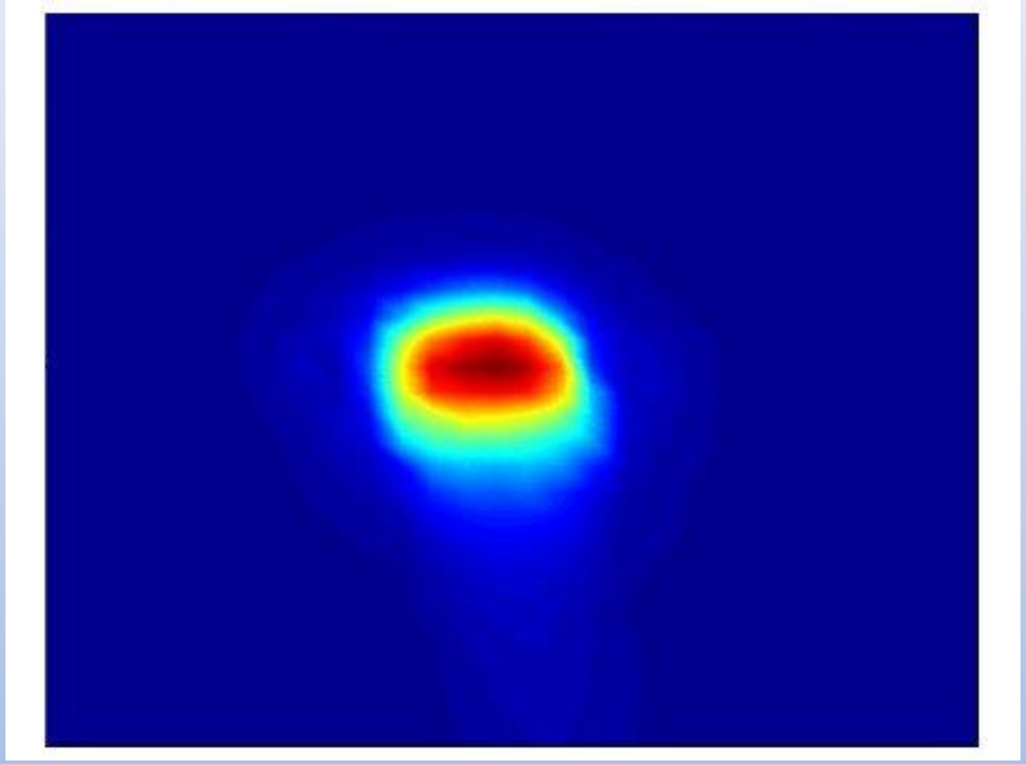
- Velocity field measured with PIV



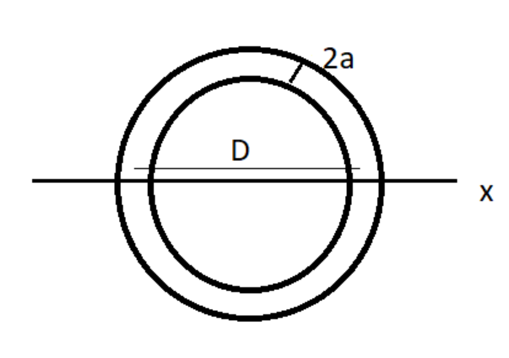
Axial velocity along the symmetry axis of the annular vortex

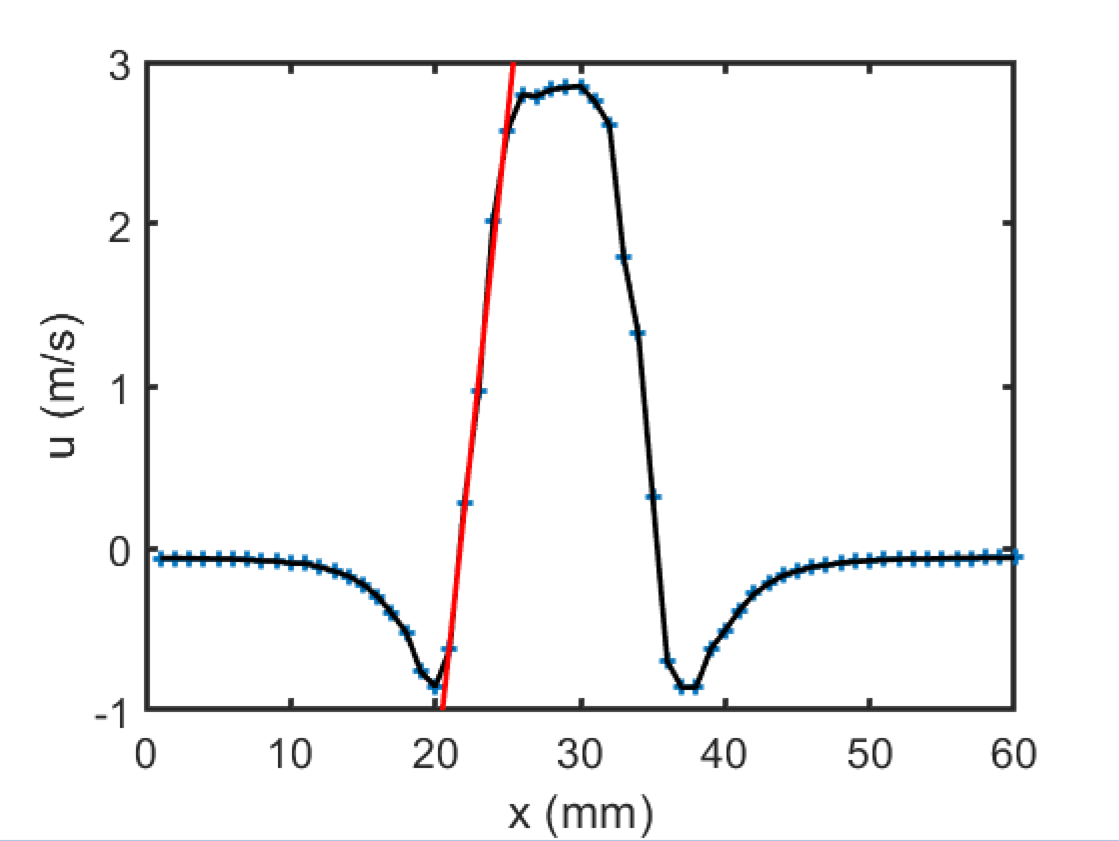
$$u_Z = \frac{\Gamma}{2}\frac{a^2}{(a + z^2)^{3/2}}$$



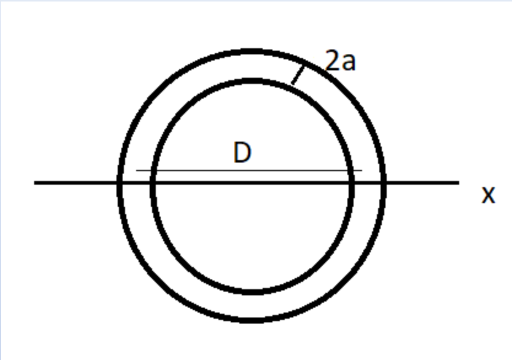
Axial velocity on a straight line passing through the center of the annular vortex.

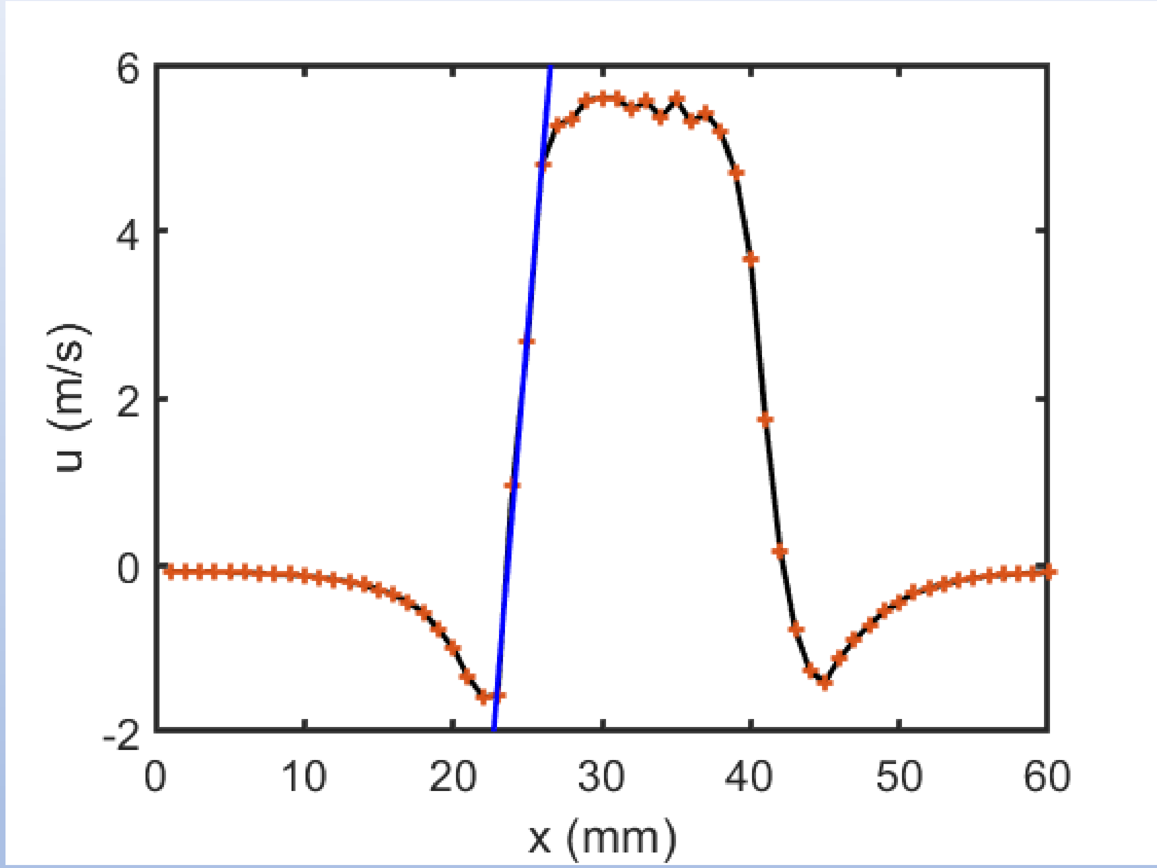




Axial velocity (u_z) vs z for an annular vortex. Re=800. The slope of the red line is 0.815 m/s/mm. The radius of the vortex core is $\sim 2.0\ mm$

Axial velocity on a straight line passing through the center of the annular vortex.





Axial velocity (u_z) vs z for an annular vortex. Re=2200. The slope of the red line is 2.209 m/s/mm. The radius of the vortex core is $\sim 1.5mm$

- If the translational speed is substracted from the velocity, the fluid particles into the vortex core follow circular paths. The acceleration of the fluid particles is $\frac{v^2}{r}$. Then, on the border of the vortex core the acceleration are:

a) 1300 m/s ^{2} for Re=800
b) 6500 m/s ^{2} for Re=2200

- As a first approximation the fluid particles in the vortex core move as in a rigid body rotation.

The accelerations into the vortex core take values between 0 and the maximum value indicated in the previous page

On the other side, in a solid body rotation the velocity difference

$$\delta V(\overline{r_2} - \overline{r_1}) \perp \overline{r_2} - \overline{r_1}$$

Then, when evaluating a longitudinal structure function, the contribution of the velocity differences inside the core vanishes

CONCLUSIONS

- The experiments we carried out recover strong accelerations as those observed in turbulent flows. Our results support the idea that strong accelerations in turbulence occur when particles are trapped inside a vortex.