Turbulent natural convection between two differentially heated vertical plates: A theoretical study

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## Outline

- Introduction
- Natural Vertical Convection
- Earlier Work
- Our Theory and Results
- Summary

### Introduction

• Thermally-driven fluid flows are ubiquitous



The solar interior includes the core, radiative zone and convective zone. The photosphere is the visible surface of the Sun. The solar atmosphere includes the chromosphere and corona.









# Natural convection between two vertical plates

- many applications in engineerir
   e.g., thermal insulation in
   double-pane windows
- ice-ocean interactions



Antarctica - credit: Pixabay



https://glassdoctor.com/expert-tរំទ្រទៃ/all-aboutwindow-glass/double-pane-windows

Fluid motion driven by both temperature difference and salinity difference between melt water and salty seawater

# How does heat transfer depend on the control parameters of the flow?

• Control parameters:

#### Rayleigh (Ra) and Prandtl numbers (Pr)

$$Ra = \frac{g\alpha H^3 \Delta}{\nu \kappa} \qquad Pr = \frac{\nu}{\kappa}$$

aspect ratios  $L_y/H$  and  $L_z/H$ 

 $\alpha$ ,  $\nu$ , and  $\kappa$  are the thermal expansion coefficient, kinematic viscosity, and thermal diffusivity of the fluid, respectively  $\Delta$  = temperature difference



• Heat transfer is measured by the dimensionless Nusselt number (Nu):

$$\mathrm{Nu} \equiv \frac{Q}{k\Delta/H}$$

which is defined as the actual heat flux Q normalized by that when there were only heat conduction

k is the thermal conductivity of the fluid

• With the Oberbeck-Bousinessq approximation, the governing equations of motion are

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \alpha g (T - T_0) \hat{z} \quad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T \quad (2)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (3)$$

#### Earlier Work (I) Laminar Vertical Convection

• Pioneer work by Batchelor 1954

#### QUARTERLY OF APPLIED MATHEMATICS

Vol. XII

October, 1954

No. 3

HEAT TRANSFER BY FREE CONVECTION ACROSS A CLOSED CAVITY BETWEEN VERTICAL BOUNDARIES AT DIFFERENT TEMPERATURES\*

> BY G. K. BATCHELOR Trinity College, Cambridge, England

• For very small Ra, Nu  $\approx 1 + a Ra^2$ 



### Earlier Work (II) Turbulent Vertical Convection

#### **Experimental studies:**



Similar results were reported for water and ethanol R.K. MacGregor and A.F. Emery, Trans. ASME, J. Heat Transfer 93, 253 (1969)



#### Chong Shen Ng<sup>1,†</sup>, Andrew Ooi<sup>1</sup>, Detlef Lohse<sup>2</sup> and Daniel Chung<sup>1</sup>

J. Fluid Mech. (2015), vol. 764, pp. 349–361 Unifying theory of thermal convection in vertical natural convection Unifying theory of thermal convection in vertical natural convection



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Direct numerical simulations of two-dimensional J. Fluid Mech. (2007), vol. 586, pp. 259–293. © 2007 Cambridge University Press doi:10.607/3002/1200700308 Printed from Unicer Kingdom in a differentially



Direct numerical simulations of two- and three-dimensional turbulent natural convection flows in a differentially *Filled Mech* (1995), vol. 304, pp. 87–118 heated cavity of aspect ratio 4. Copyright © 1995 Cambridge University Press

> F. X. TRIAS, M. SORIA, A. OLIVA and C. D. PÉREZ-SEGARRA

Centre Tecnologic **Director** 11 **PP PIPE CP CONTENT Basic riscol Contract Simulations of view o-dimensional** doi:10.1017/S002211200700808 Primed in the United Kingdom (Received 15 Ecbruary 2006 and in revised form 18 April 2007) A set of complete two-a **Charoctic Contractors Contractors** 

An overview of the numerical algo code and the simulations is presented time-averaged flow structure, the **0** gw of a set of selected monitoring points, balances and the internal waves motion

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• DNS data (Howland et al., JFM 2022)



Pr = 5

overlap layer does not exist for  $Pr \ge 5$ 

• DNS data (Howland et al., JFM 2022)

$$Nu^{-3/4}(RaPr)^{1/4} = \frac{K}{2}\ln(NuRaPr) + 2A(Pr) + 2B$$
 (HH2005)



Scaling theory is not supported by DNS data

e wind-based Reynolds number scaling, Re, is defined as be (a,b), we find that the trends predicted by (3.9) for  $\langle \varepsilon_{\mu'} \rangle_{bulk}$  and h the Apotheratheoretical attemption bulk dissipation (3.5), with the the the second with a second with the second with the second se xtend the Great and (3.9) need to be els for  $U_{bulk}/U$ ,  $M_{integral of the bulk dissipation of the mean, i.e.$  $u_{lk}$ , in terms of Re, Ra, Nu and Pr. Chong Shen  $Ng^{1+}$ , Andrew Ooi<sup>1</sup>, Detlef Lohse<sup>2</sup> and Daniel Chung<sup>1</sup> Chong Shen Ng<sup>1,+</sup>, Andrew Ooi<sup>1</sup>, Detlef Lohse<sup>2</sup> and Daniel Chung<sup>1</sup> J. Fuild Mech. (2013), vol. 764, pp. 349-361 <sup>1</sup>Department of Mechanicengleening (201 5) jversity of 64e bpune 34 2 to 36 1910, Qustan Bridge University I 3. Glabal Fais Charges in Josephine in the state churcher Fais sipations and MESA+ Institute, University of Twente, 7500 AE Enschede, The Netherlands RB at However, while a Ravie is the Bénard convection, the Jation the Structure is the Bénard Reare not closed exact relations involving Nu and Reare not closed approximately approxima applicability of the Grossmann-Lohse (GL) theory, which was originally doveloped for horizontal natural (Rayleigh Bénard) and ection. In accordance with the GL theory, it is shown that the boundary-layer thicknesses of the vetokity and te Abertature fields in vertical fatural the normalised mean boundary-layer thicknesses scale with the is the velocity verof monthead the maximum mean velocity. Away from the walls, the dissipation  $\langle wT \rangle = f_w - \frac{1}{dissipation} full of the GL theory, is found to be the Kolmpson of the GL theory, is found to be the Kolmpson of the Kolmpson of the GL theory, is found to be the Kolmpson of the Kolmp$ The prientation of this flow preceived d22 distrange 14 crowised 2 nNovember 2014 accepted

#### Our Theory Emily S.C. Ching, Phys. Rev. Fluids 8, L022601 (2023)

- Focus on the large aspect-ratio limit
- Using Reynolds decomposition

 $w(x, y, z, t) = W(x) + w'(x, y, z, t) \qquad T(x, y, z, t) - T_0 = \Theta(x) + \theta'(x, y, z, t)$ and taking time average of the equations of motion  $\frac{d}{dx} \langle u \\ \frac{d}{dx} \langle u'w' \rangle_t = \nu \frac{d^2}{dx^2} W + \alpha g \Theta \qquad \text{mean momentum balance equation}$  $\frac{d}{dx} \langle u'w' \rangle_t = \kappa \frac{d^2}{dx^2} \Theta \qquad \text{mean thermal energy balance equation}$ 

- W(0)=W(H/2)=0,  $\Theta(0)=\Delta/2$  and  $\Theta(H/2)=0$
- $Q = -k d \Theta/dx |_{x=0}$
- Mean flow equations are not closed

Integrating the mean momentum equation gives

$$\langle u'w'\rangle_t = \nu \frac{dW}{dx} - u_\tau^2 + \alpha g \int_0^x \Theta(x')dx'$$

• Evaluating the result at  $x = x_0$ , where

 $u_{\tau}^2 = \alpha g \int_0^{x_0} \Theta(x) dx$ 

$$\nu dW/dx|_{x=x_0} = \langle u'w' \rangle_t(x_0)$$

$$\Theta(x/\delta_T) = \Delta \Phi(\xi)/2$$

$$\xi_0 = x_0/\delta_T$$
  
 $\delta_T$  is the thermal boundary layer  
thickness defined by Nu  $\equiv$  H/2 $\delta_T$ 

$$\operatorname{Re}_{\tau}^{2}\operatorname{NuPrRa}^{-1} = \frac{1}{4}\int_{0}^{\xi_{0}} \Phi(\xi)d\xi \equiv I(\operatorname{Ra},\operatorname{Pr})$$

#### Evaluate $\Phi(\xi)$ and $\xi_0$ using DNS data by Howland et al., JFM 930, A32 (2022)



 $\Phi(\xi)$  approaches a Pr-dependent asymptotic form as Ra increases

•  $\xi_{0}$  has a weak dependence on Ra for each Pr



Pr = 1 (circles), 2 (squares), 5 (diamonds),10 (triangles) and 100 (inverted triangles)

- These results lead us to make the assumption  $I(\text{Ra}, \text{Pr}) \rightarrow f(\text{Pr})$  in the high-Ra limit
- This yields the first relationship

 $\operatorname{Re}_{\tau}^{2}\operatorname{NuPrRa}^{-1} = f(\operatorname{Pr})$  high-Ra limit

 $\frac{1}{100} \frac{1}{100} \frac{1}$ End of the distance of the compensated plots while the full of the decident of the second of the sec  $y = ax^{\text{inandene}}$ , x = 0, x =Tespect to x at  $\xi = \frac{1}{24}$  gives  $x = \frac{1}{24}$  bives Stress  $\pm 0$  and the heat  $\frac{11}{110}$  No  $\frac{1}{200}$  therefore expanses on 100 and  $\pm d\Theta/dx|_{x=0}$ . a Pizzo/u and we mand we take in all the period of the state ear the wall the molecular diffuse view of the another take the characteristic lenge to be  $l = v/u_1$  for  $P = 0 \gg 1$  and  $l = \kappa 4\partial_2$  for  $P = \kappa 4\partial_2$  for to betail the the sector and the the  $\frac{1}{2} = \frac{1}{2} + \frac{1$ PR (C8) for 92 8 deteco) the highlighter interest a consent of the former of the state of the st ationstructures in the second states of the second

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### Summary

- We have carried out a theoretical study of large aspectratio turbulent vertical convection
- Our analysis is based on the mean flow equations and yields two relationships between heat flux (Nu) and wall shear stress ( $Re_{\tau}$ ).
- These two relationships give the dependence of Nu and  $\operatorname{Re}_{\tau}$  on Ra and Pr in the high-Ra limit and our theoretical results are in excellent agreement with the direct numerical simulation data for  $\operatorname{Pr} \geq 1$ .
- Our work in progress shows that the there are two contributions to the Reynolds number (Re) measuring the maximum mean vertical velocity and they have different dependence on Ra.

#### Thank you for your attention!