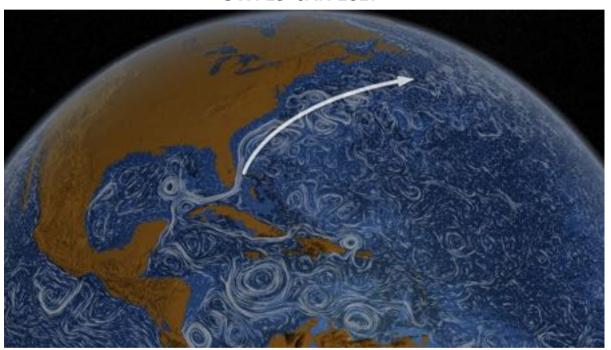
## TURBULENT AT HIGH AND LOW ROTATION RATES: EULERIAN AND LAGRANGIAN STATISTICS



Luca Biferale Dept. Physics University of Rome 'Tor Vergata' & INFN, Italy UWI 19° JAN 2017





F.Bonaccorso, I.Mazzitelli (Rome, Italy) M.Hinsberg, F. Toschi (Eindhoven, The Netherlands) A.Lanotte (Lecce, Italy) S. Musacchio (Nice, France) P.Perlekar (Hydebarad, India)





PRACE 09\_2256 ROTATING TURBULENCE 2015 – 55MH

## WHERE/WHAT IS THE PROBLEM?

•Too many turbulences?

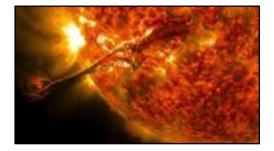
- •Can we disentangle universal from non-universal properties?
- •Can we understand universal properties ?
- •Does 'computing' mean 'understanding'? (Computo ergo sum?)

•Can we use computation to make experiments that cannot be done on a lab?

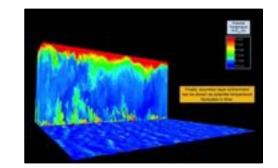




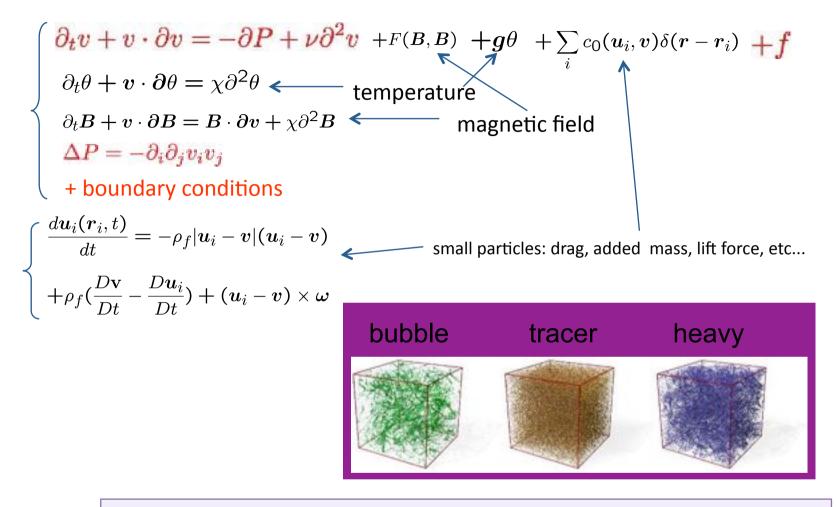








### **Turbulence or Turbulences?**



Flows with additives:

Advection-diffusion-reaction of passive scalar/vectors (temperature, magnetic field, chemical reactions, etc...) Advection-diffusion of active scalars/vectors (convection, magnetic dinamo)

Polymers (drag reduction)

Bubbles/Droplets (two phase flows, rain formation, etc...)

Swimmers (cooperative hydrodynamical interactions)



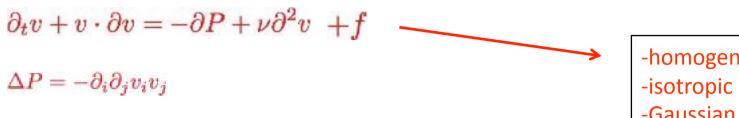
Leonardo da Vinci (~ 1500): "doue la turbolenza (turbulence) dell'acqua <u>si</u> <u>genera (is produced)</u>; doue la turbolenza dell'acqua <u>si mantiene (is transferred)</u>; doue la turbolenza dell'acqua <u>si posa (is dissipated)</u>"

**R.P. Feynman (1970):** "Certainly. I've spent years trying to solve some difficult problems without success. The theory of turbulence is one. In fact, <u>it is still</u> <u>unsolved</u>."

**J. Von Neumann (1949)** "[...] The entire experience with the subject indicates that the purely analytical approach is <u>beset with difficulties</u>, <u>which at the moment</u> are prohibitive. [...] Under these conditions there may be some hope to "<u>break the deadlock</u>" by extensive, but **well-planned** computational efforts.

**Sir H. Lamb (1932):** "I am an old man now, and when I die and go to Heaven there are two matters on which I hope enlightenment. One is quantum electrodynamics (QED) and the other is turbulence of fluids. About the former, I am really rather optimistic."

## THE HYDROGEN ATOM OF TURBULENCE

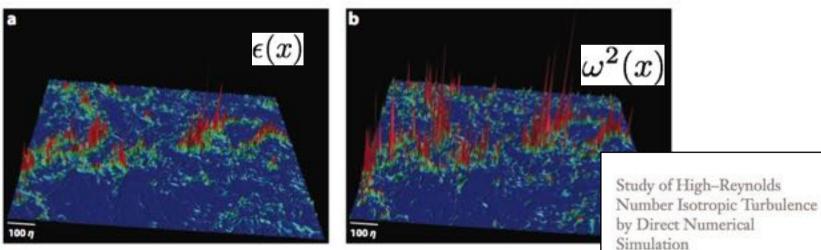


+ periodic boundary conditions



## HOMOGENEOUS TURBULENCE

**3D CASE: MAINLY UNSOLVED!** 



#### Figure 4

Simulation Takashi Ishihara,<sup>1</sup> Toshiyuki Gotoh,<sup>2</sup>

and Yukio Kaneda<sup>1</sup>

er of Geopoteniesal Science and Engreening, Graduate School of Engree Napus University, Chikow ku, Napus 464 8601, Jupan, email: shiftaraffest support sur-jp Department of Kineside and Engineering Vendation, Graduan-School of Engineering Support Institute of Technology, Editors, News Int, Napore 406-8733, Japan

Snapshot of the intensity distributions of (a) the energy-dissipation rate  $\tilde{e} = e/(2v)$  and (b) the enstrophy  $\Omega = \omega^2/2$  on a DNS-ES at  $R_1 = 675$  in arbitrary units.





#### Entry #: 84174

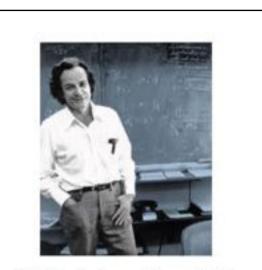
### Vortices within vortices: hierarchical nature of vortex tubes in turbulence

Kai Bürger<sup>1</sup>, Marc Treib<sup>1</sup>, Rüdiger Westermann<sup>1</sup>, Suzanne Werner<sup>2</sup>, Cristian C Lalescu<sup>3</sup>, Alexander Szalay<sup>2</sup>, Charles Meneveau<sup>4</sup>, Gregory L Eyink<sup>2,3,4</sup>

<sup>1</sup> Informatik 15 (Computer Graphik & Visualisierung), Technische Universität München
 <sup>2</sup> Department of Physics & Astronomy, The Johns Hopkins University
 <sup>3</sup> Department of Applied Mathematics & Statistics, The Johns Hopkins University

- <sup>4</sup> Department of Mechanical Engineering, The Johns Hopkins University

# NAVIER-STOKES 3D-2D



"With turbulence, it's not just a case of physical theory being able to handle only simple cases-we can't do any. We have no good fundamental theory at all." (Feynman, 1979, Omni Magazine, Vol. 1, No.8).

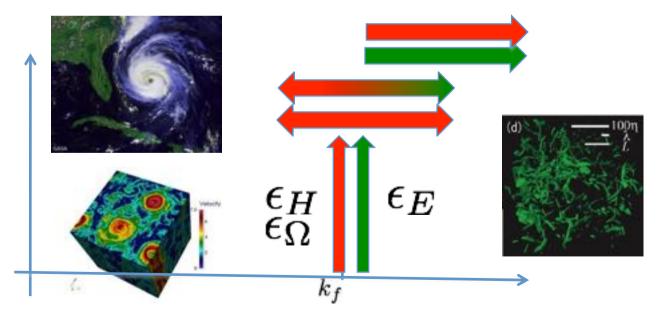
# MOTIVATIONS:

A TALE ABOUT TRANSFER PROPERTIES OF INVISCID CONSERVED QUANTITIES, KINETIC ENERGY, HELICITY ENSTROPHY, MAGNETIC HELICITY ETC...

Q1: HOW TO PREDICT THE DIRECTION OF THE TRANSFER (FORWARD/BACKWARD) AND ITS ROBUSTNESS UNDER EXTERNAL PERTURBATION (FORCING/BOUNDARY CONDITIONS)?

Q2: HOW MUCH THE FLUCTUATIONS AROUND THE MEAN TRANSFER ARE INTENSE AND SELF-SIMILAR (INTERMITTENCY AND ANOMALOUS SCALING) ?

AS A MATTER OF FACT, FOR 3D NAVIER STOKES EQUATIONS, WE DO NOT KNOW HOW TO PREDICT NEITHER THE SIGN OF THE MEAN ENERGY TRANSFER NOR THE INTENSITY OF THE FLUCTUATIONS AROUND IT.

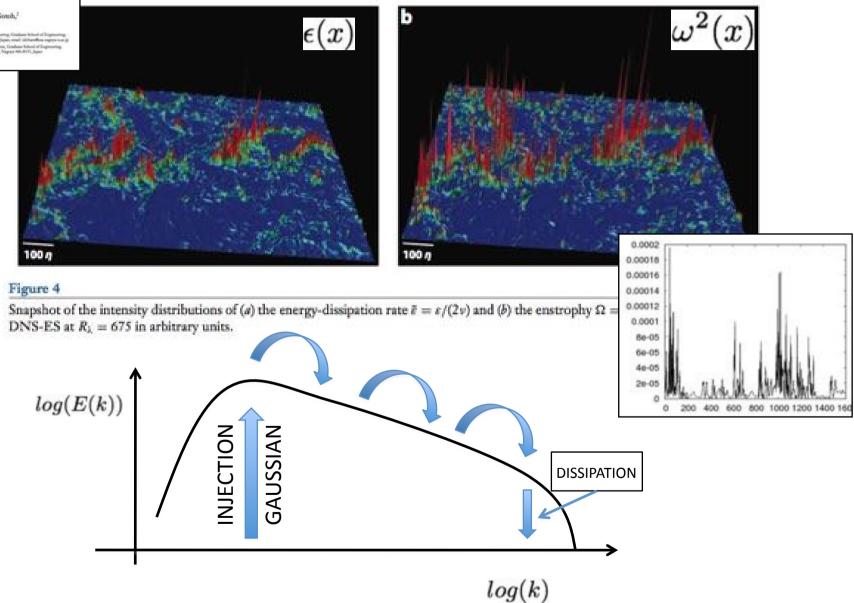


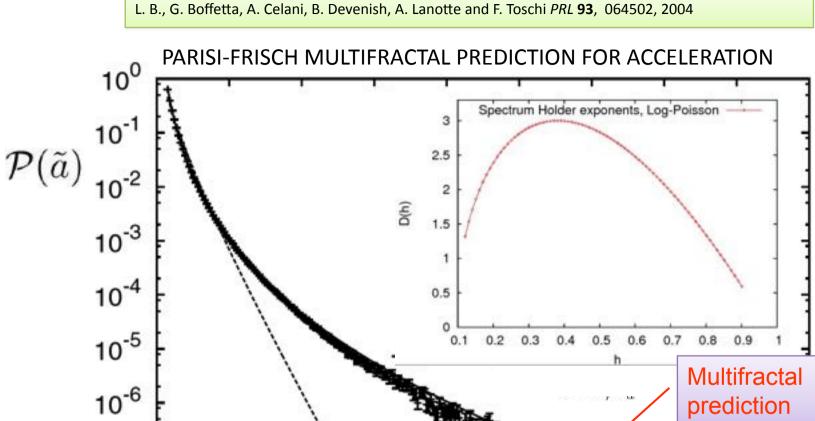
#### Study of High–Reynolds Number Isotropic Turbulence by Direct Numerical Simulation

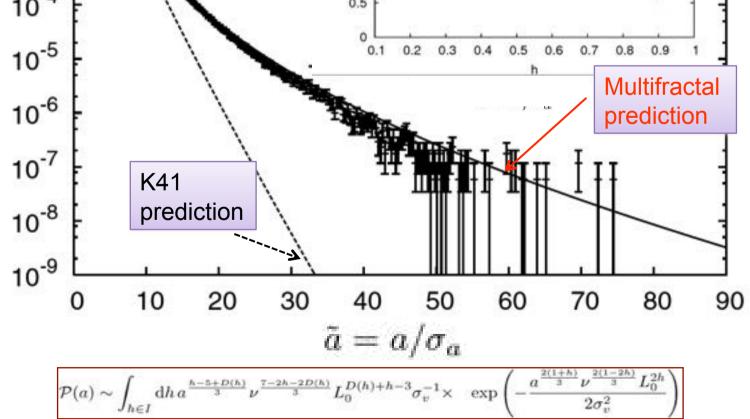
#### Takashi Ishihara,<sup>1</sup> Toshiyuki Gotoh,<sup>2</sup> and Yukio Kaneda<sup>1</sup>

<sup>1</sup>Depresent of Comparison Network and Expresenting, Conducto Moleci of Expresenting, Nappy Determing, Chilawa ia, Nappy 144, 4001, Jupan, cond. Mithandhean support and gi "Department of Microsoft and Explaneting Homolation, Canadam Medica of Expresenting, Nappy Institute of Thelesedage, Chilana, Shoren Ka, Nappy 140-4533, Jupan

## 3D HOMOGENEOUS AND ISOTROPIC TURBULENCE FLUCTUATIONS: SMALL-SCALES INTERMITTENCY







- MOTIVATION: WHY ROTATING TURBULENT FLOWS ARE IMPORTANT

- DIRECT AND INVERSE ENERGY TRANSFERS (2D-3D PHYSICS)

- OUR DNS (DIFFERENCES WRT PREVIOUS STUDIES)

- EULERIAN STATISTICS (MEAN SPECTRAL PROPERTIES)

- EULERIAN STATISTICS (LARGE FLUCTUATIONS)

- LAGRANGIAN STATISTICS (EFFECTS OF CORIOLIS AND CENTRIFUGAL FORCES)

- LAGRANGIAN STATISTICS (SINGLE PARTICLE DISPERSIONS)

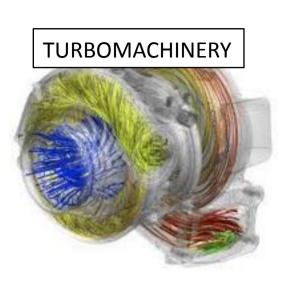
- CONCLUSIONS

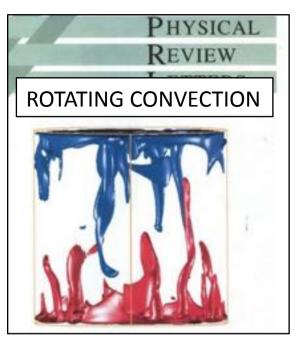
## TAYLOR-COUETTE

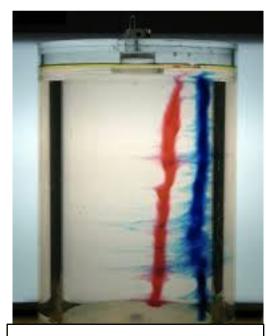
ROTATING CONVECTION (+ STRATIFICATION + MHD)

## ROTATING RAYLEIGH-TAYLOR

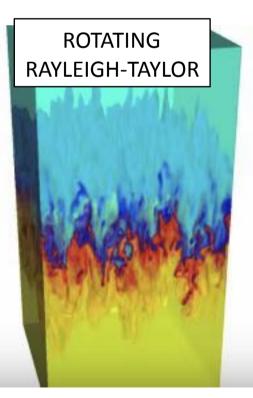
Recent reviews/books by Lohse, Boffetta, Cambon, Clercx, Davidson etc...

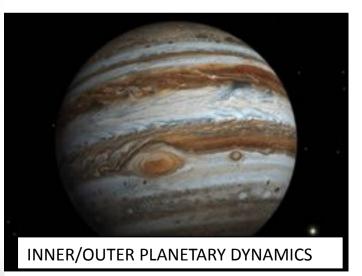






CYCLONIC-ANTICYCLONIC DYN.





NAVIER\_STOKES EQS IN A ROTATING FRAME (NO BOUNDARIES)

DNS: A. Pouquet, P. Mininni, A. Alexakis, S. Chen, G. Eyink ....

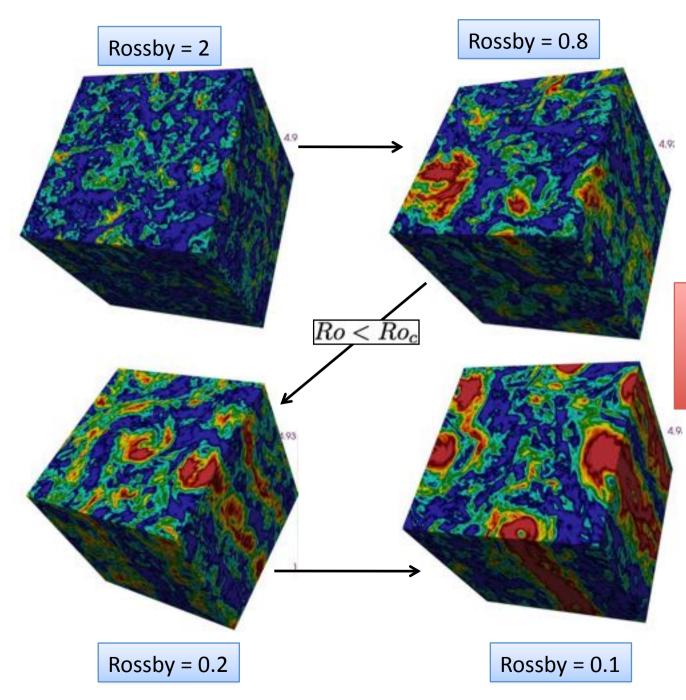
$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \mathbf{\Omega} \times \mathbf{v} = \nabla P + \nu \nabla^2 \mathbf{v} + \mathbf{F} - \alpha \mathbf{v}$$

$$oldsymbol{\Omega}$$
 =rotation  $P=P_0+rac{1}{2}|oldsymbol{\Omega} imes {f r}|^2$ 

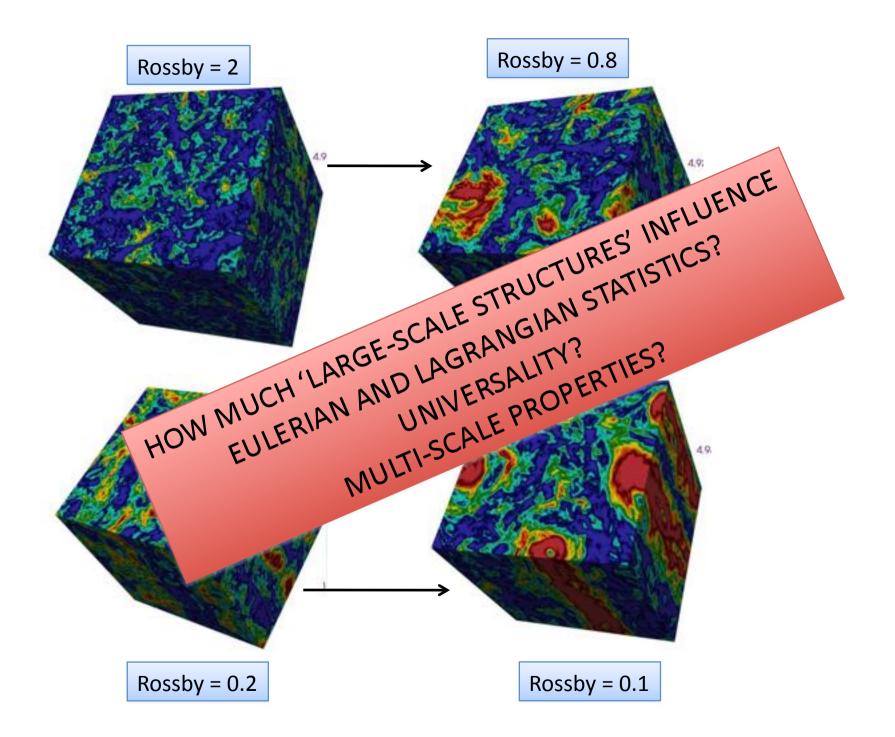
**F**-large scale Forcing  $\alpha = \text{large scale energy sink}$ 

ROSSBY NUMBER ~ NON-LINEAR/ROTATION

$$\begin{array}{l} \operatorname{Ro} \sim \frac{v_0}{\Omega \, L_0} \\ \operatorname{Ro} \geq Ro_c \rightarrow & \text{forward energy transfer} \\ \operatorname{Ro} \leq Ro_c \rightarrow & \text{forward energy transfer} \end{array}$$



HOMOGENEOUS ANISOTROPIC 2D & 3D PHYSICS CHOERENT -STRUCTURES



#### OUR DNS DATA-BASE (EULERIAN + LAGRANGIAN)

**NEW FEATURES:** 

- 1) IDEAL FORCING MECHANISM (AS NEUTRAL AS POSSIBLE: ISOTROPIC; NON HELICAL, TIME-COLORED) + LARGE SCALE FRICTION
- 2) UNPRECEDENTED NUMERICAL RESOLUTION/SCALE SEPARATION (UP TO 4096^3)
- 3) LAGRANGIAN STATISTICS (MILLIONS OF TRACERS AND INERTIAL PARTICLES)

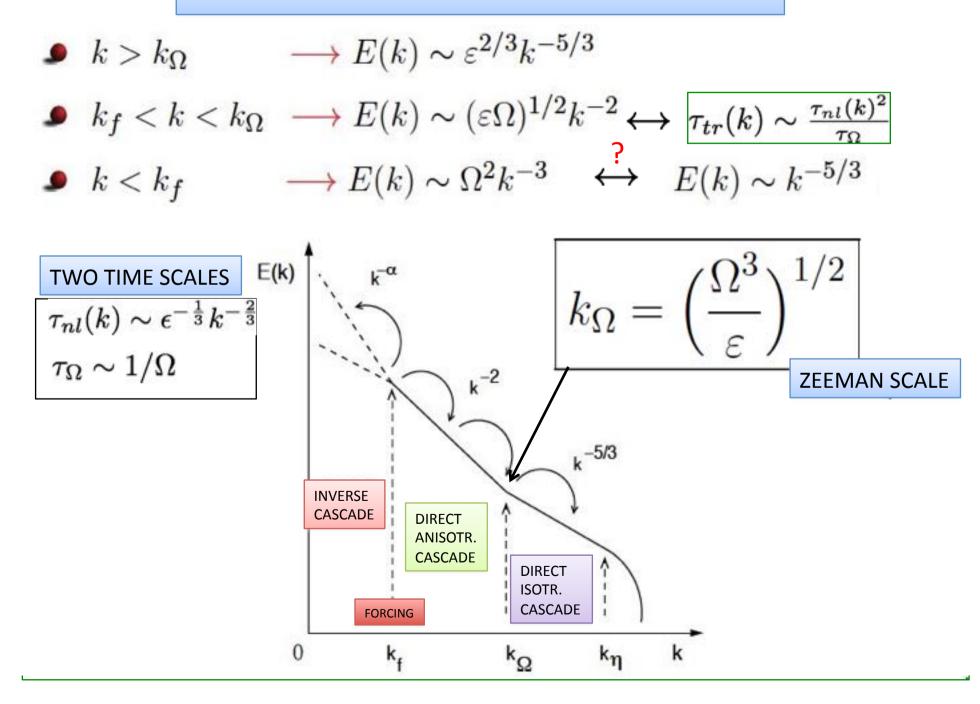
N	Ω	ν	e	€f	$u_0$	$\eta/dx$	$\tau_\eta/dt$	$Re_{\lambda}$	Ro	$f_0$	$\tau_{f}$	$T_0$	α
1024	4	$7 \times 10^{-4}$	1.2	1.2	1.05	0.67	120	150	0.78	0.02	0.023	0.17	0.0
1024	10	$6 \times 10^{-4}$	0.46	0.59	1.6	0.76	294	580	0.24	0.02	0.023	0.25	0.1
2048	4	$2.8 \times 10^{-4}$	1.2	1.2	1.05	0.67	380	230	0.76	0.02	0.023	0.17	0.0
2048	10	$2.2 \times 10^{-4}$	0.45	0.64	1.7	0.72	550	1170	0.25	0.02	0.023	0.3	0.1
4096	10	$1 \times 10^{-4}$	0.46	0.65	1.7	0.78	1010	1600	0.25	0.02	0.023	0.3	0.1

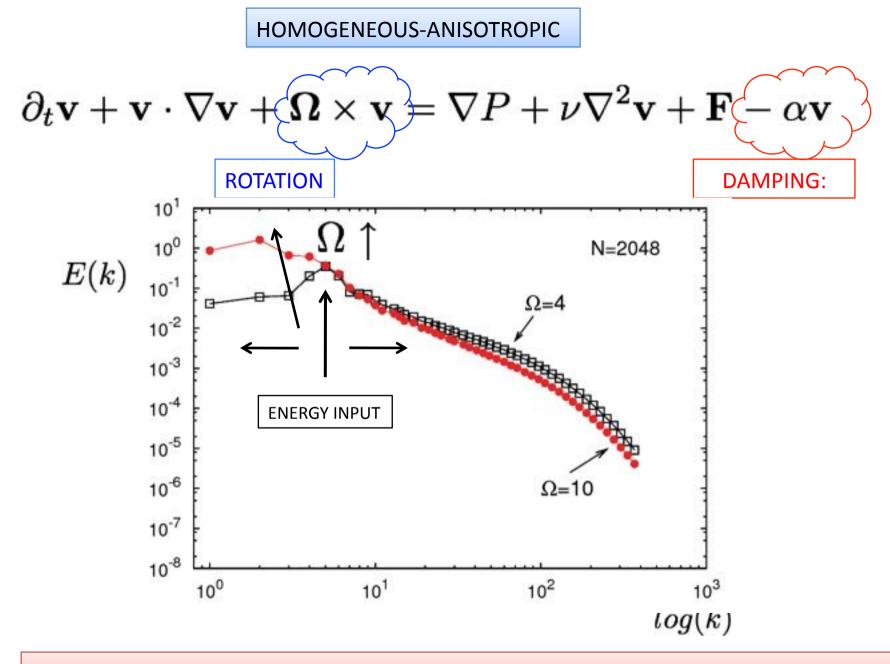
TABLE I: Eulerian dynamics parameters. N: number of collocation points per spatial direction;  $\Omega$ : rotation rate;  $\nu$ : kinematic viscosity;  $\epsilon = \nu \int d^3x \sum_{ij} (\nabla_i u_j)^2$ : viscous energy dissipation;  $\epsilon_f = \int d^3x \sum_i f_i u_i$ : energy injection;  $u_0 = 1/3 \int d^3x \sum_i u_i^2$ : mean kinetic energy;  $\eta = (\nu^3/\epsilon)^{1/4}$ : Kolmogorov dissipative scale;  $dx = L_0/N$ : numerical grid spacing;  $L_0 = 2\pi$ : box size;  $\tau_\eta = (\nu/\epsilon)^{1/2}$ : Kolmogorov dissipative time;  $Re_\lambda = (u_0\lambda)/\nu$ : Reynolds number based on the Taylor micro-scale;  $\lambda = (15\nu u_0^2/\epsilon)^{1/2}$ : Taylor micro-scale;  $Ro = (\epsilon_f k_f)^{1/3}/\Omega$ : Rossby number defined in terms of the energy injection properties, where  $k_f = 5$  is the wavenumber where the forcing is acting;  $f_0$ : intensity of the Ornstein-Uhlenbeck forcing;  $\tau_f$ : decorrelation time of the forcing;  $T_0 = u_0/L_0$ : Eulerian large-scale eddy turn over time;  $\alpha$ : coefficient of the damping term  $\alpha\Delta^{-1}u$ .

MAX RESOLUTION

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \mathbf{\Omega} \times \mathbf{v} = \nabla P + \nu \nabla^2 \mathbf{v} + \mathbf{F} - \alpha \mathbf{v}$$

#### DIMENSIONAL PHENOMENOLOGY





FORCING: 2°-order OU-PROCESSS: ISOTROPIC, HOMOGENEOUS NOT DELTA-CORRELATED

# 1 WHAT ARE THE INTERACTIONS/MECHANISMS RESPONSIBLE FOR THE INVERSE ENERGY CASCADE, 2D-3D?

2. WHAT ABOUT THE SMALL-SCALES VELOCITY STATISTICS IN PRESENCE OF A LARGE SCALE INVERSE ENERGY TRANSFER: EFFECTS OF CHOERENT VORTEX STRUCTURES Commun. Math. Phys. 115, 435-456 (1988)

## The Beltrami Spectrum for Incompressible Fluid Flows

Peter Constantin 1.\* and Andrew Majda 2.\*\*

#### The nature of triad interactions in homogeneous turbulence

Fabian Walette Center for Turbalence Research, Stanford University-NASA Ames, Building 500. Stanford, California 94305-3030

(Received 24 July 1991; accepted 22 October 1991)

$$u(k) = u^+(k)h^+(k) + u^-(k)h^-(k)$$

$$egin{aligned} m{h}^{\pm} &= \hat{m{
u}} imes \hat{m{k}} \pm i \hat{m{
u}} \ \hat{m{
u}} &= m{z} imes m{k} / ||m{z} imes m{k}||. \end{aligned}$$

$$i\mathbf{k} imes \mathbf{h}^{\pm} = \pm k\mathbf{h}^{\pm}$$

$$\begin{cases} E = \sum_{k} |u^{+}(k)|^{2} + |u^{-}(k)|^{2}; \\ H = \sum_{k} k(|u^{+}(k)|^{2} - |u^{-}(k)|^{2}). \end{cases}$$

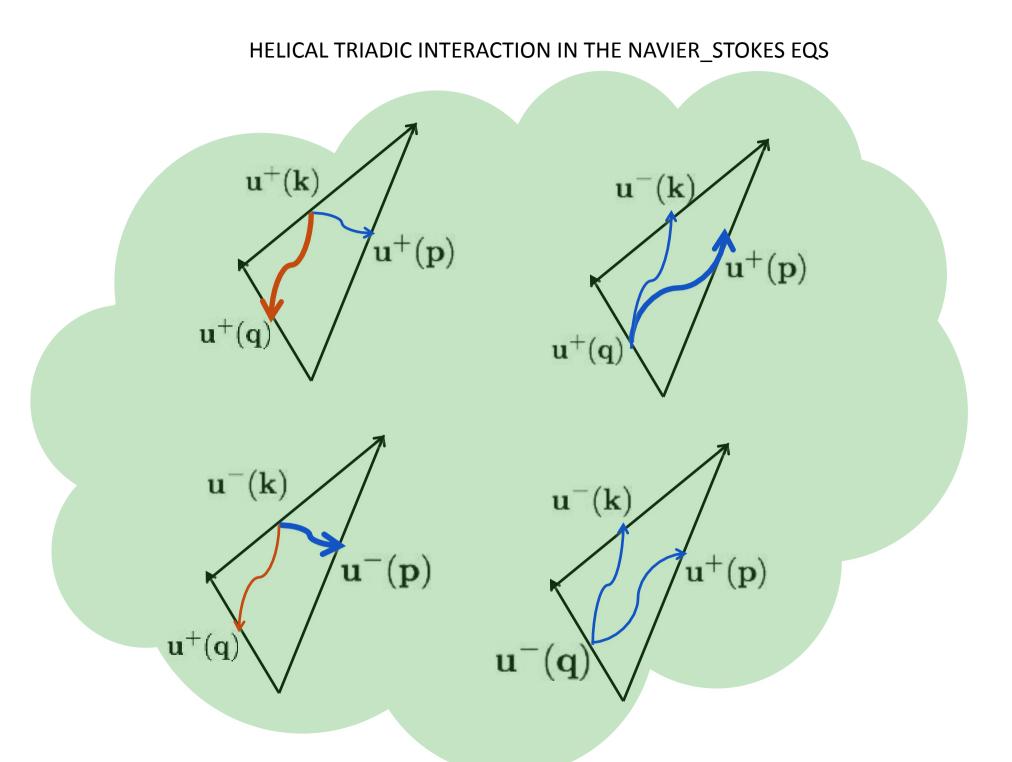
$$u^{s_k}(\mathbf{k},t) \quad (s_k = \pm 1)$$

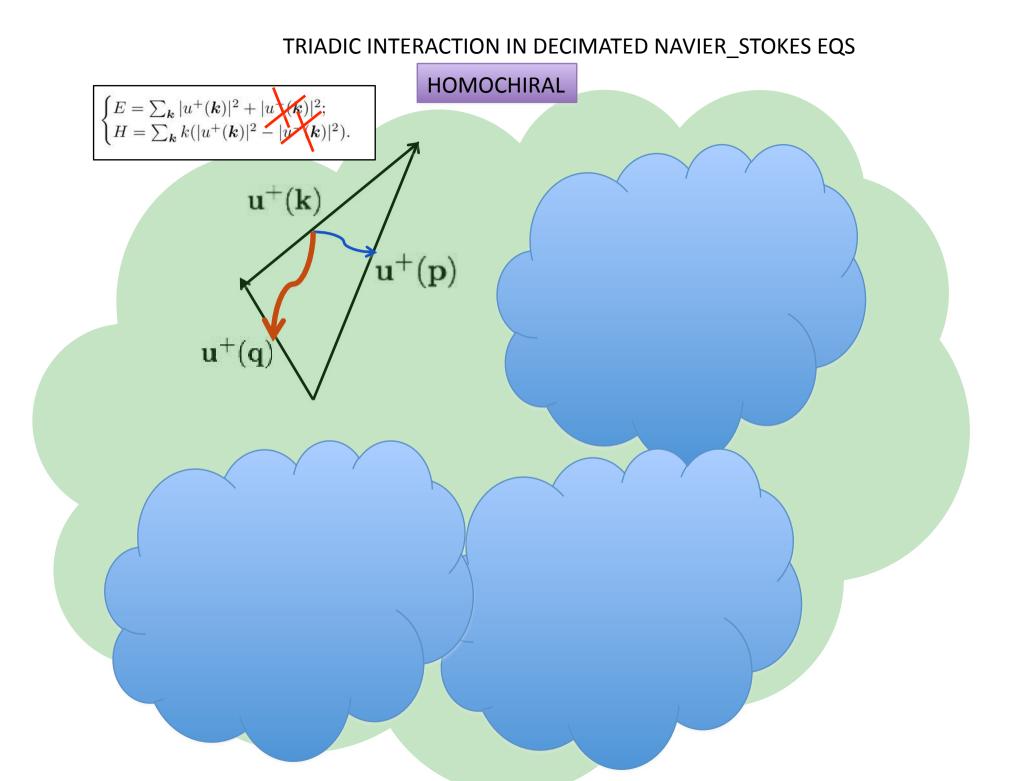
$$\frac{d}{dt}u^{s_k}(\mathbf{k}) + \nu k^2 u^{s_k}(\mathbf{k}) = \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0} \sum_{s_p,s_q} g_{\mathbf{k},\mathbf{p},\mathbf{q}}(s_p p - s_q q)$$

$$\times [u^{s_p}(\mathbf{p})u^{s_q}(\mathbf{q})]^*. \quad (15)$$

Eight different types of interaction between three modes  $u^{s_k}(\mathbf{k})$ ,  $u^{s_p}(\mathbf{p})$ , and  $u^{s_q}(\mathbf{q})$  with  $|\mathbf{k}| < |\mathbf{p}| < |\mathbf{q}|$  are allowed according to the value of the triplet  $(s_k, s_p, s_q)$ 

$$\begin{split} \dot{u}^{s_{k}} &= r(s_{p}p - s_{q}q) \frac{s_{k}k + s_{p}p + s_{q}q}{p} (u^{s_{p}}u^{s_{q}})^{*}, \\ \dot{u}^{s_{p}} &= r(s_{q}q - s_{k}k) \frac{s_{k}k + s_{p}p + s_{q}q}{p} (u^{s_{q}}u^{s_{k}})^{*}, \\ \dot{u}^{s_{q}} &= r(s_{k}k - s_{p}p) \frac{s_{k}k + s_{p}p + s_{q}q}{p} (u^{s_{k}}u^{s_{p}})^{*}. \end{split}$$





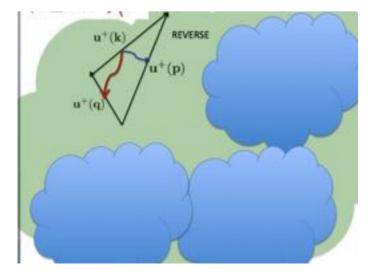
HOMOCHIRAL 3D NAVIER STOKES EQS.

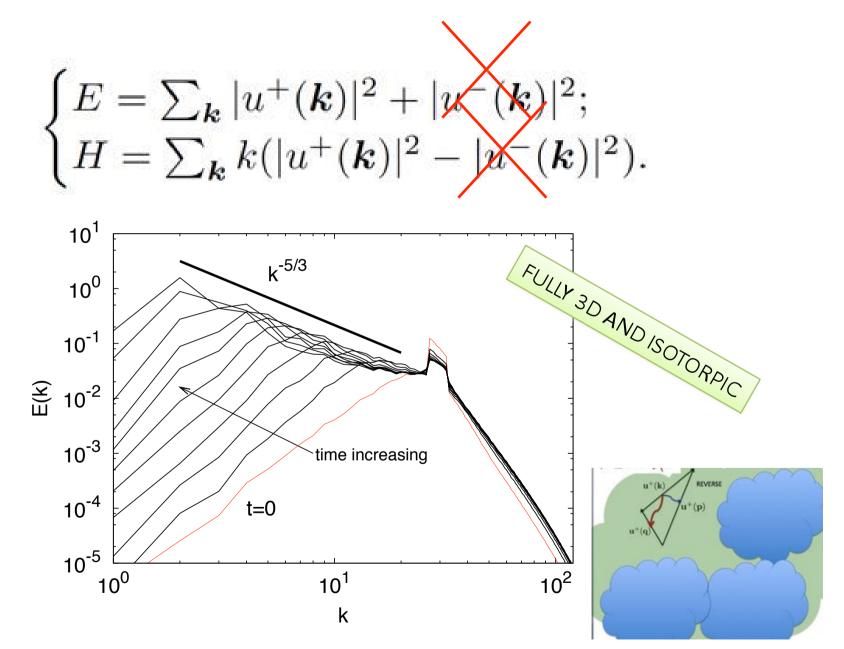
$$\mathcal{P}^{\pm} \equiv rac{h^{\pm} \otimes \overline{h^{\pm}}}{\overline{h^{\pm}} \cdot h^{\pm}}.$$
  $v^{\pm}(x) \equiv \sum_{k} \mathcal{P}^{\pm} u(k);$   
 $u(k) = u^{+}(k)h^{+}(k) + u^{-}(k)h^{-}(k)$ 

LOCAL BELTRAMIZATION (IN FOURIER)

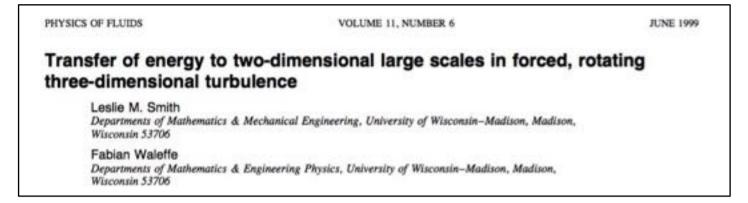
$$\partial_t v^+ + \mathcal{P}^+ B[v^+, v^+] = \nu \Delta v^+ + \mathbf{f}^+$$

decimated-NSE





L.B., S. MUSACCHIO & F. TOSCHI Phys. Rev. Lett. 108 164501, 2012.

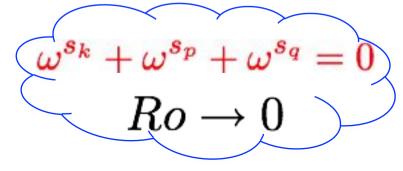


$$\mathbf{u}(\mathbf{k}) = u^{+}(\mathbf{k})e^{+it\omega^{+}(\mathbf{k})}\mathbf{h}^{+}(\mathbf{k}) + u^{-}(\mathbf{k})e^{+it\omega^{-}(\mathbf{k})}\mathbf{h}^{-}(\mathbf{k})$$

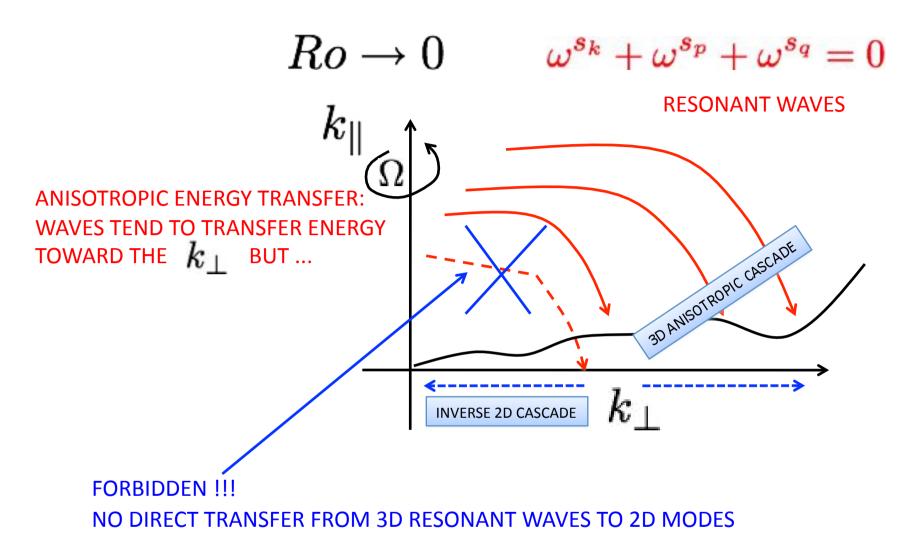
$$i\mathbf{k} \times \mathbf{h}^{\pm} = \pm k\mathbf{h}^{\pm} \qquad \omega^{\pm}(\mathbf{k}) = \pm 2\Omega\frac{k_{z}}{k}$$

$$\frac{d}{dt}u^{s_{k}}(\mathbf{k}) + \nu k^{2}u^{s_{k}}(\mathbf{k}) = \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0}\sum_{s_{p},s_{q}}g_{\mathbf{k},\mathbf{p},\mathbf{q}}(s_{p}p - s_{q}q)$$

$$e^{i(\omega^{s_{k}} + \omega^{s_{p}} + \omega^{s_{q}})t/Ro} \times [u^{s_{p}}(\mathbf{p})u^{s_{q}}(\mathbf{q})]^{*}. \qquad (15)$$

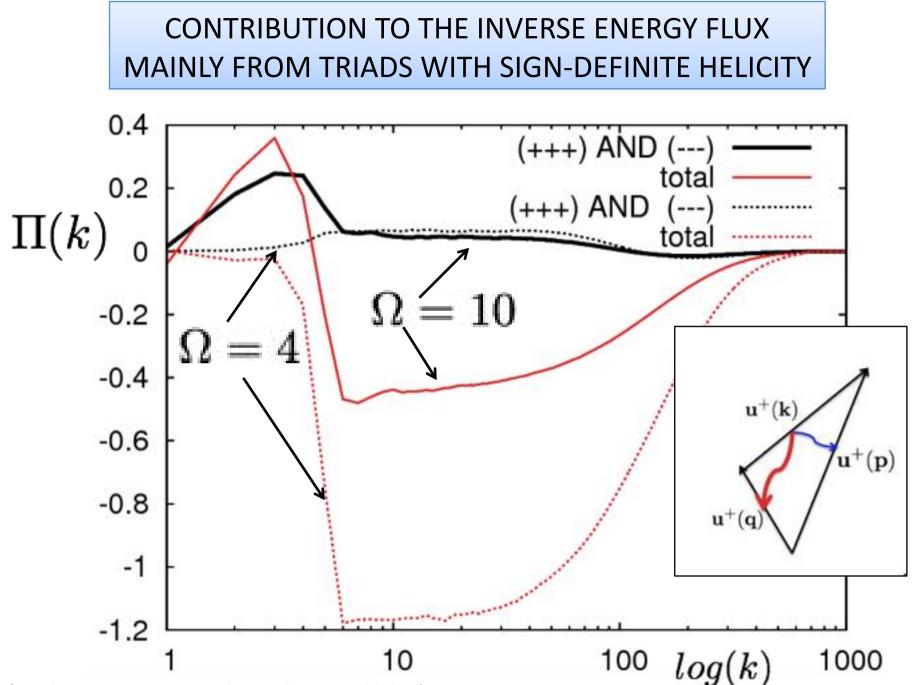


VVAVE-INTERACT

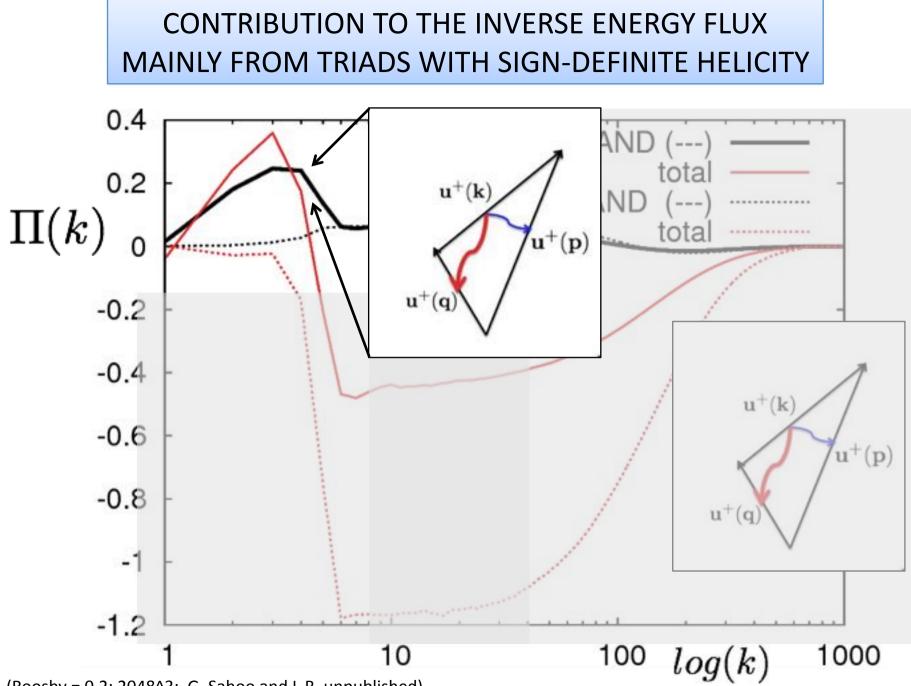


THERE EXISTS A BUFFER REGION IN THE K-SPACE CLOSE TO THE 2D MODES WHERE TRIADIC RESONANT WAVES ARE LESS AND LESS EFFICIENT:

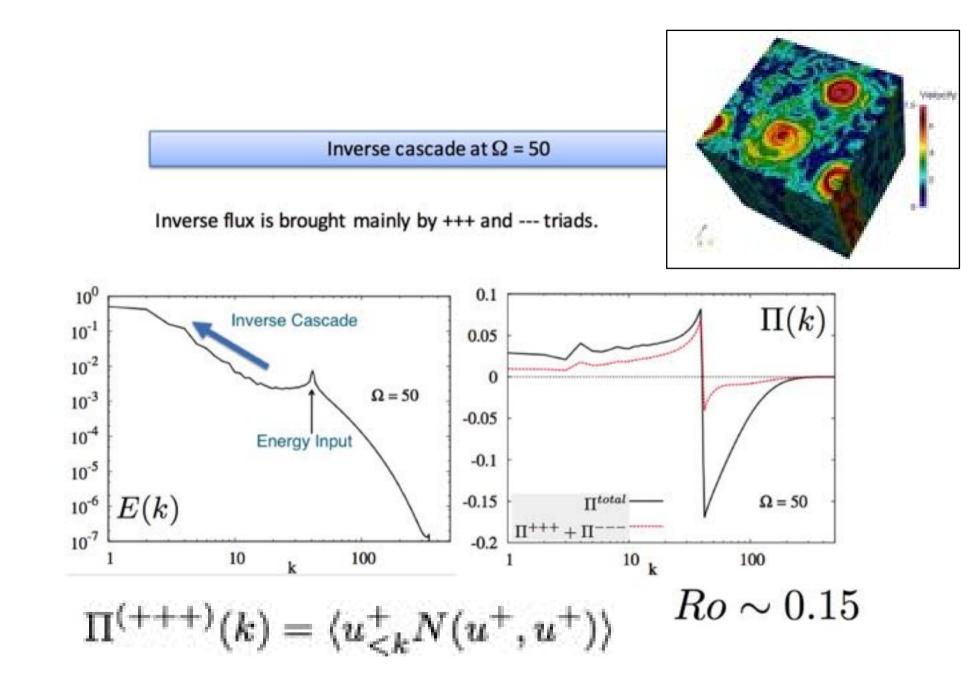
- -) O(Ro) INTERACTIONS
- -) QUARTET-INTERACTIONS
- -) TURBULENCE



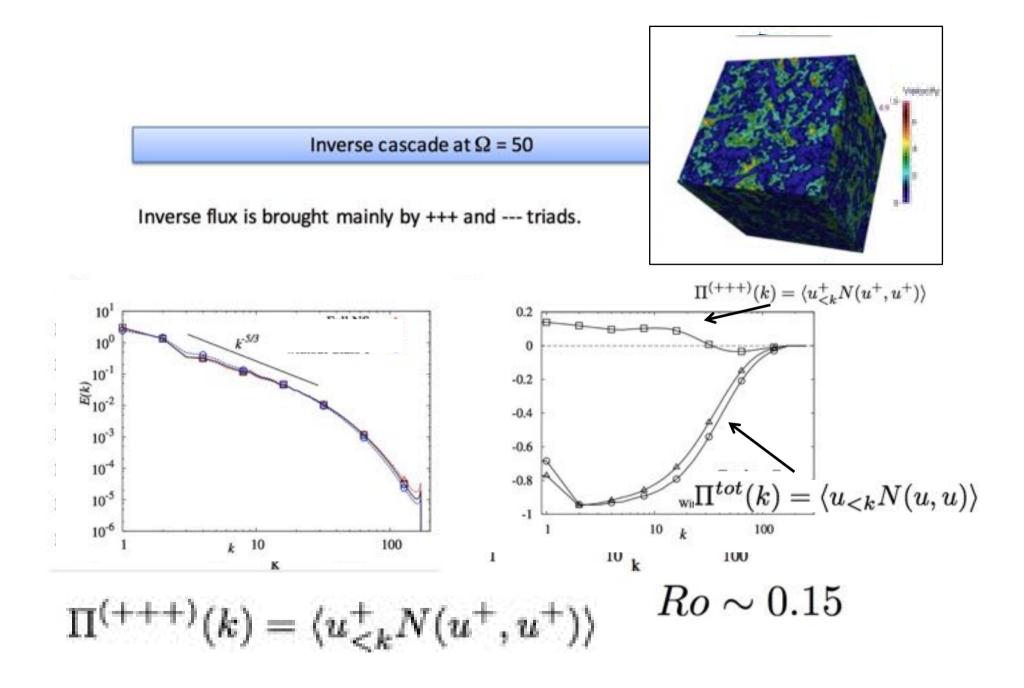
(Roosby = 0.2; 2048<sup>3</sup>; G. Sahoo and L.B, unpublished)



(Roosby = 0.2; 2048<sup>3</sup>; G. Sahoo and L.B, unpublished)



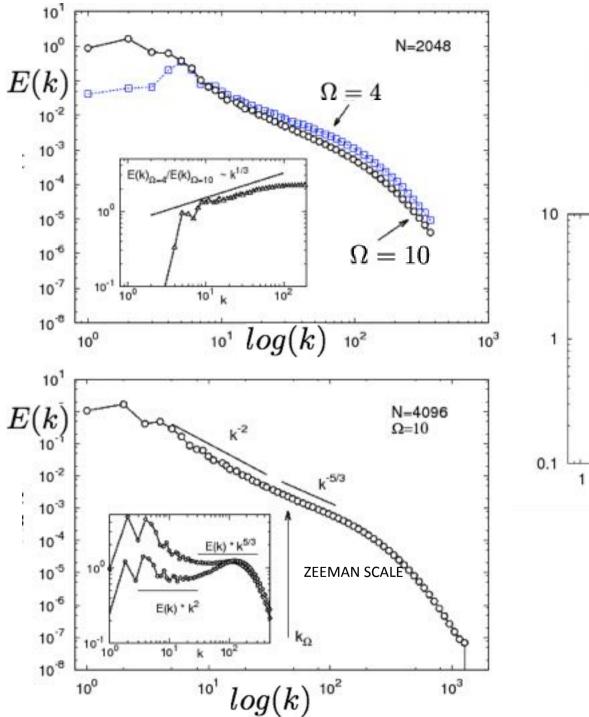
WITH G. SAHOO AND P. PERLEKAR (unpublished)

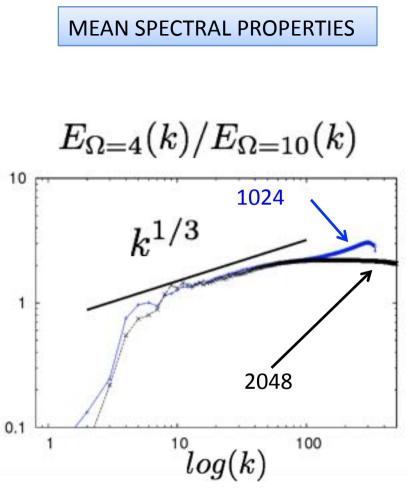


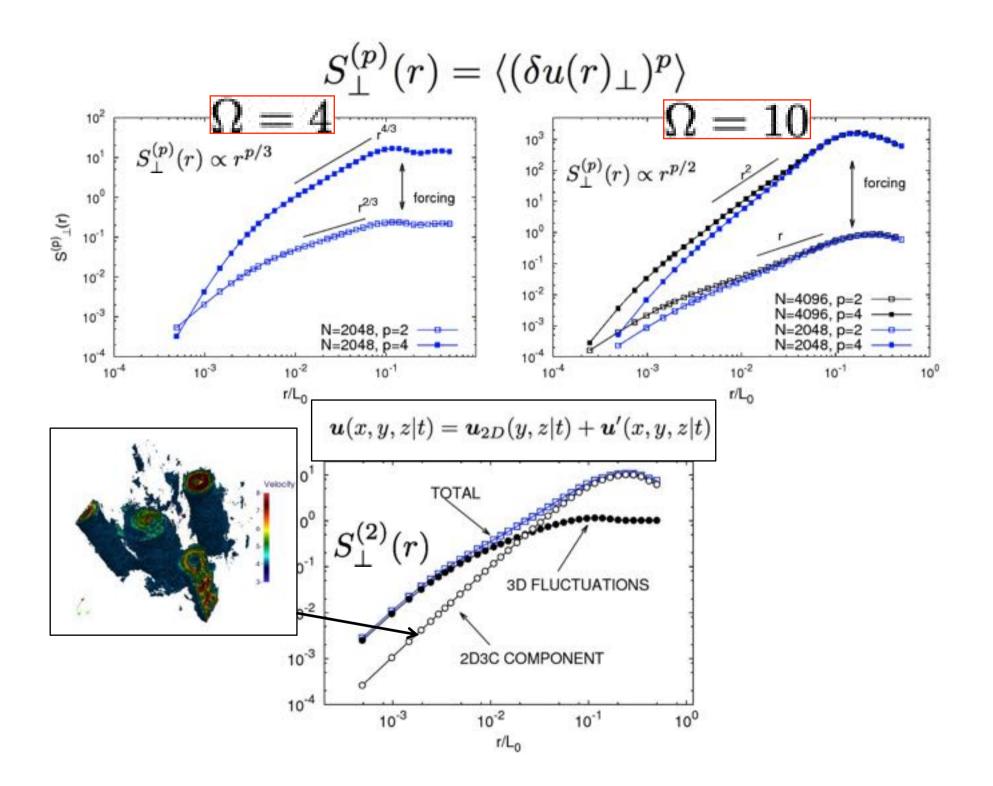
WITH G. SAHOO AND P. PERLEKAR (unpublished)

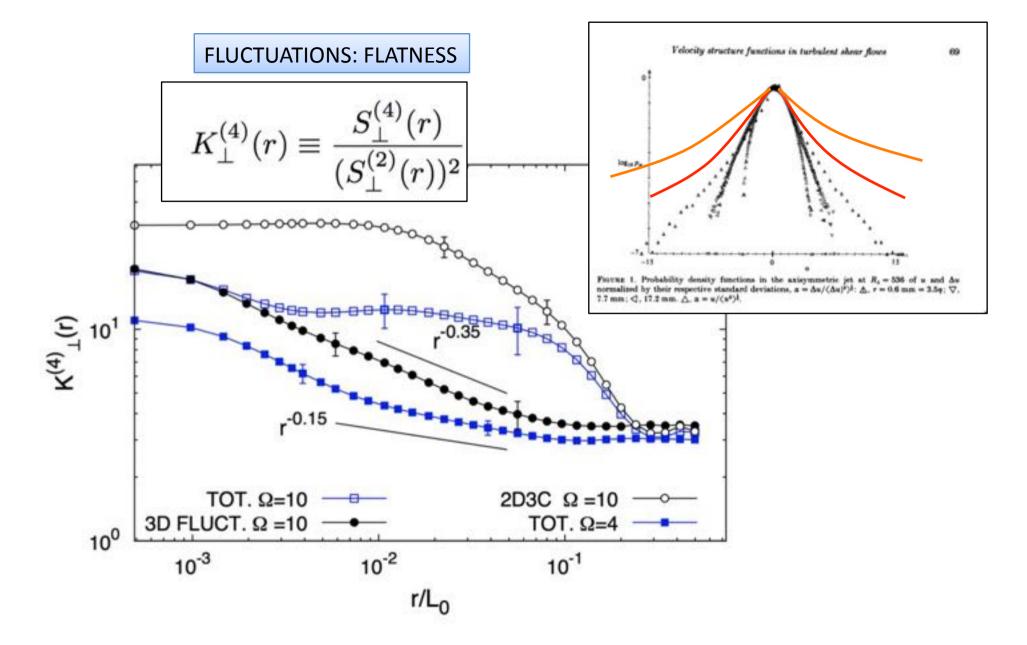
1 WHAT ARE THE INTERACTIONS/MECHANISMS RESPONSIBLE FOR THE INVERSE ENERGY CASCADE, 2D-3D?

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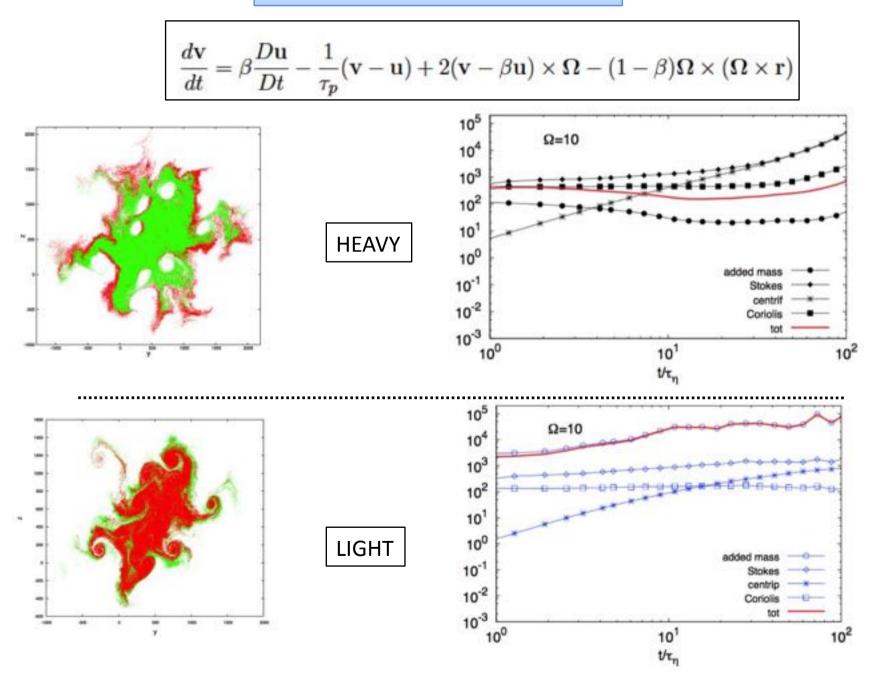




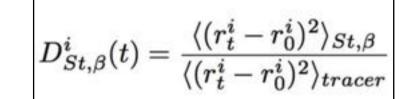


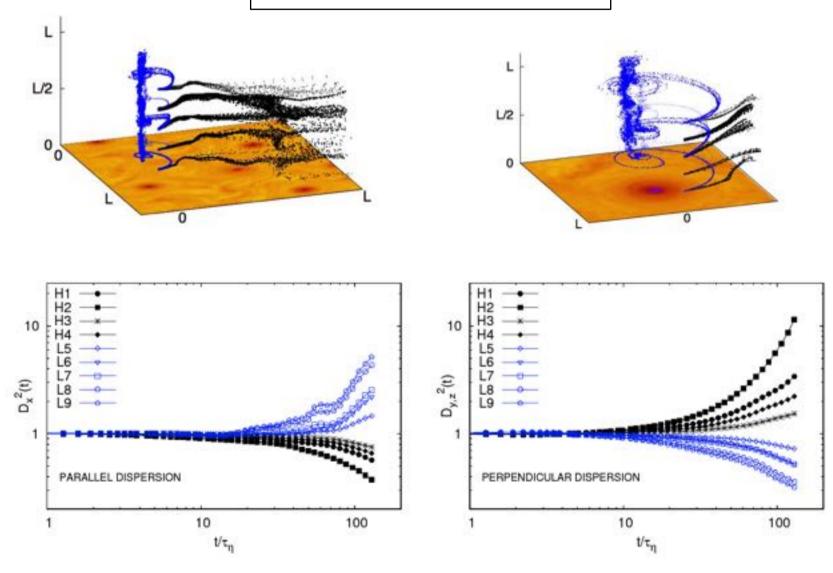
- NON-GAUSSIAN PROPERTIES DEPEND ON THE WAY YOU DECOMPOSE THE FIELD - AFTER FILTERING THE 2D3C COMPONENT: SCALING PROPERTIES ARE BACK (BUT NOT HIT!)

### **RMS FORCES ALONG TRAJECTORIES**



### INERTIA: SINGULAR EFFECT ON SINGLE PARTICLE DISPERSION





CONCLUSIONS

-HIGH RESOLUTION ROTATING TURBULENCE: FIRST ATTEMPT TO CONTROL SIMULTANEOUSLY EULERIAN & LAGRANGIAN STATISTICS

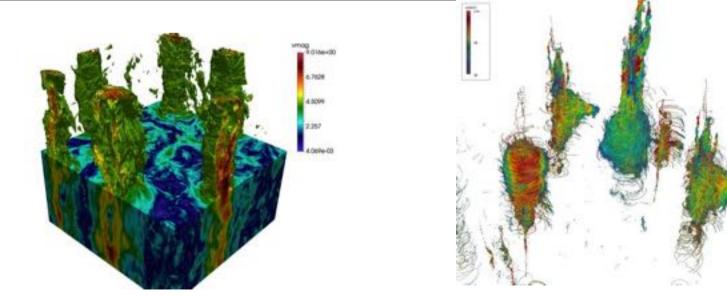
-IDEAL SET-UP (1): HOMOGENEOUS AND ISOTROPIC TIME-COLORED FORCING

-IDEAL SET-UP (2): SCALE-SEPARATION

-STRONG INFLUENCE OF LARGE-SCALE (NON-UNIVERSAL?) VORTICAL STRUCTURES

-DEPARTURE FROM GAUSSIANITY (DEPENDING ON HOW YOU MEASURE IT: 2D3C-3D3D)

-EFFECTS OF LARGE-SCALE STRUCTURES ON PARTICLES' DISPERSION



L.B., F. Bonaccorso, I. Mazzitelli, A. Lanotte, S. Musacchio, P. Perlekar, F. Toschi and M. Hinsberg, PRX 6 041036 (2016)