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Turbulence modification by inertial particles

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Turbulence modification by inertial particles

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Dipartimento di Ingegneria Meccanica e Aerospaziale
Sapienza Università di Roma

NewTURB MEETING
Tor Vergata, July 2016

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Particle laden flows

Nature & Technology

- Liquid and solid fuels

  [Jenny et *al.* Prog. Comb. Sci. (2012)]

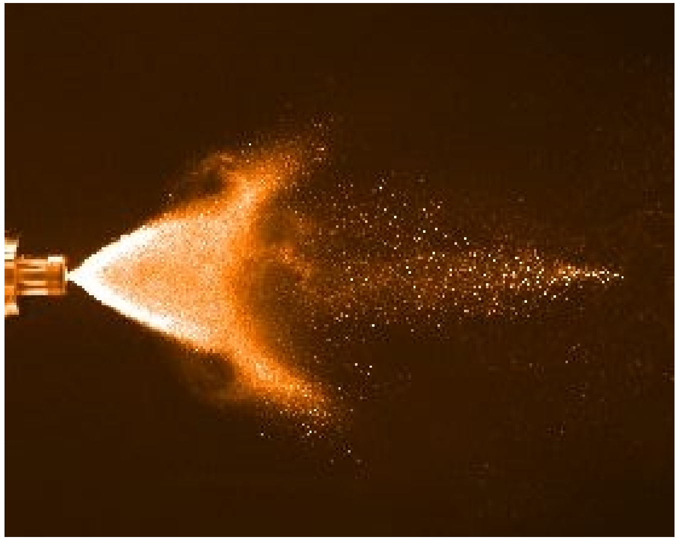
- Rain formation in clouds

  [Falkovich *al.* Nat. (2002)]

- Cyclonic separators

  [Kilstrom, Patent No. 5,935,279. (1999)]

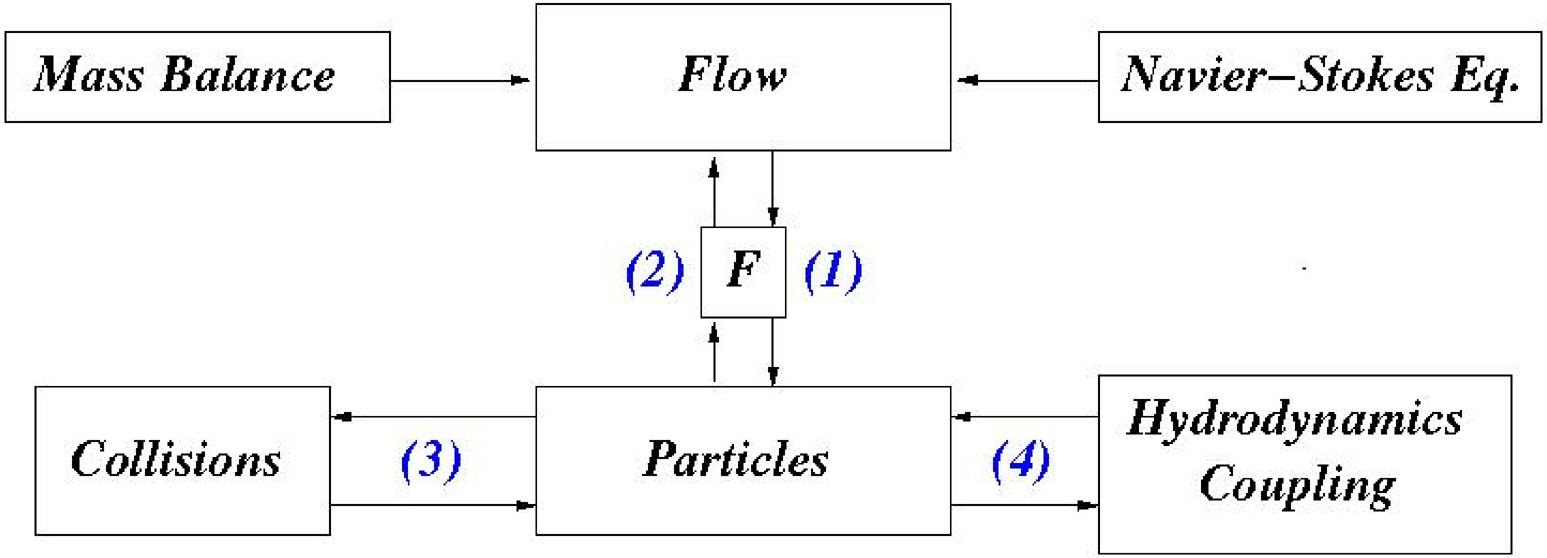
ICE injector



Vacuum cleaner

Storm on mount Amiata, Tuscany

Fluid/disperse-phase interaction



[Elghobashi (1994); Balachandar & Eaton (2010)]

- (1) one-way coupling
- (1) + (2) two-way coupling
  [dilute suspension $\Phi_V \ll 1$ but finite mass loading $\Phi = \rho_p/\rho_f \, \Phi_V$]
- (1) + (2) + (3) + (4) four-way coupling
- beyond the one-way coupling $\cdots\cdots$
  $\Rightarrow$ turbulence modulation in the *two-way coupling regime*

Particle In Cell approach (PIC) [Crowe et al. *J. Fluid Eng.* (1977)]

- Eulerian description (fluid) & Lagrangian tracking (particles) [Eaton (2009); Balachandar & Eaton (2010); Elghobashi (1994)]

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \pi + \nu \nabla^2 \mathbf{u} - \frac{1}{\rho_f} \sum_p \mathbf{D_p(t)} \delta \left[\mathbf{x} - \mathbf{x_p(t)} \right]$$

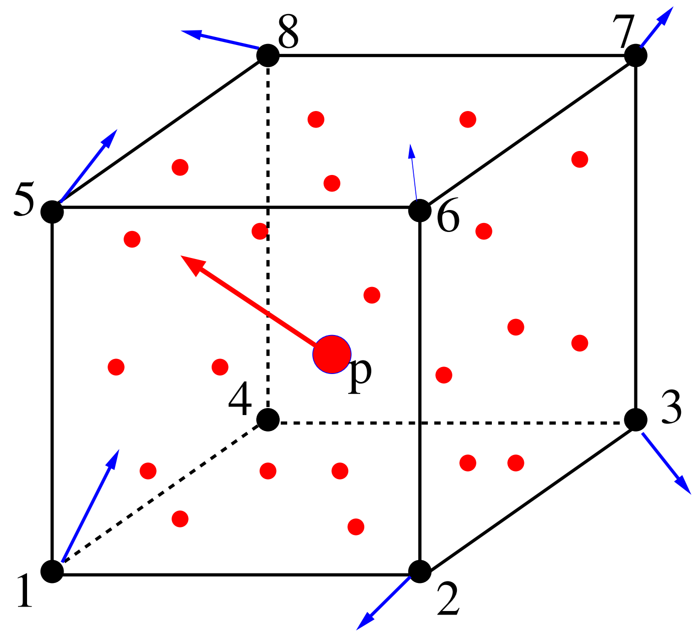
- The *back-reaction* **F** is singular: average on the cell ΔV_{cell}

$$\mathbf{F}(\mathbf{x}_p) = \frac{1}{\Delta V_{cell}} \frac{1}{\rho_f} \mathbf{D}_p$$

equivalent to $\{\mathbf{F}(\mathbf{x}_q)\}_{q=1,8}$

$$\sum_{q=1}^{8} \mathbf{F}(\mathbf{x}_q) = \mathbf{F}(\mathbf{x}_p)$$

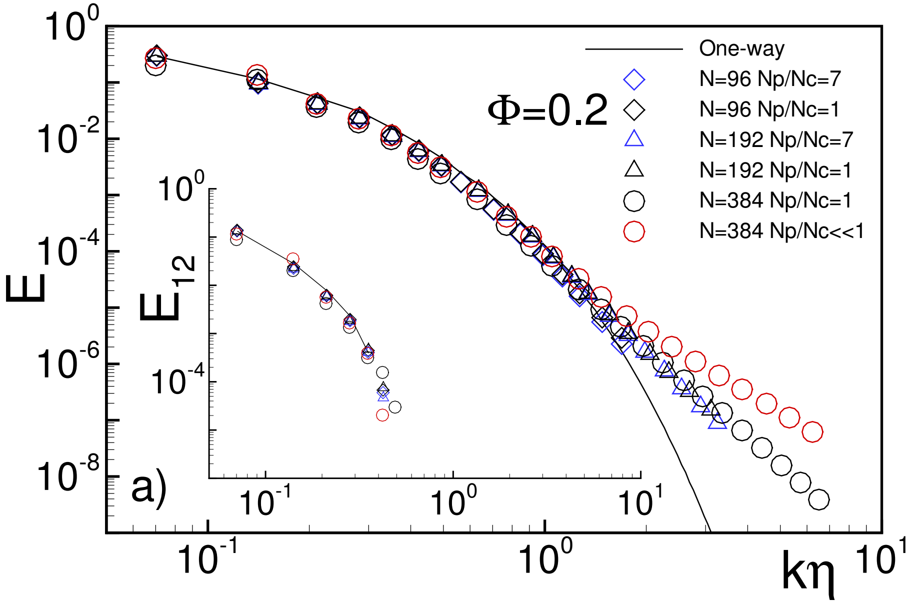
$$\sum_{q=1}^{8} (\mathbf{x}_q - \mathbf{x}_p) \times \mathbf{F}(\mathbf{x}_q) = 0$$

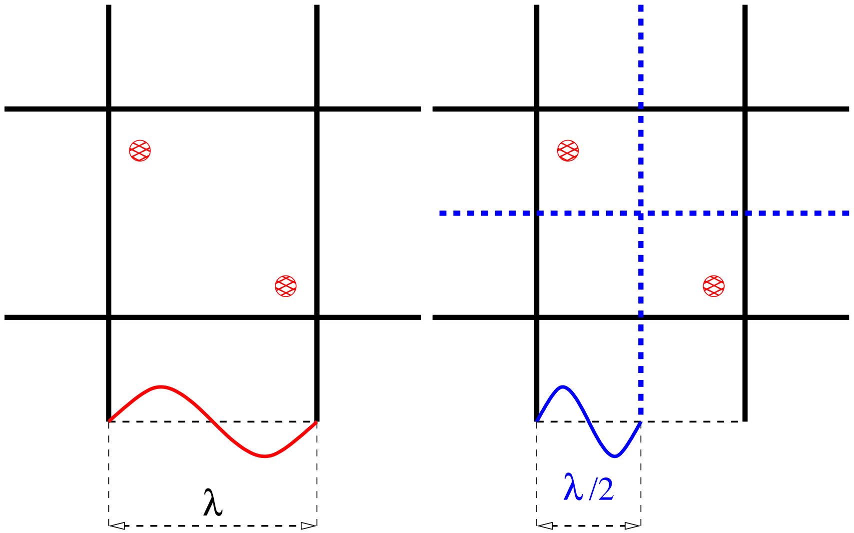


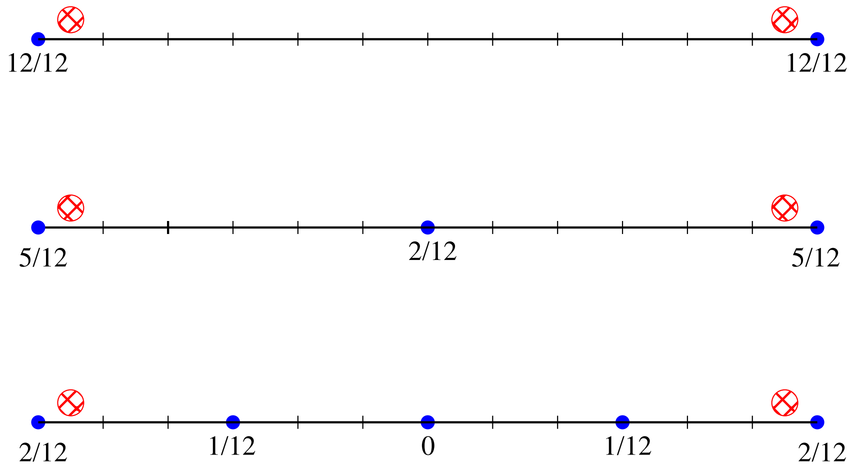
PIC: numerical issues

The tails of the spectra hardly decay when $N_p/N_c \ll 1$

- fine grids require a large number of particles
- grid dependent forcing
- limitations on the mass load





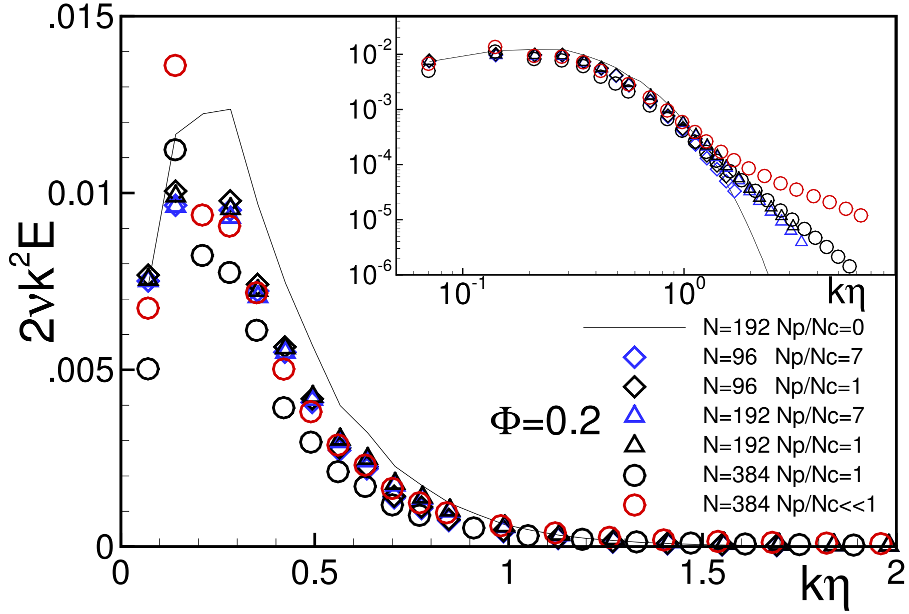


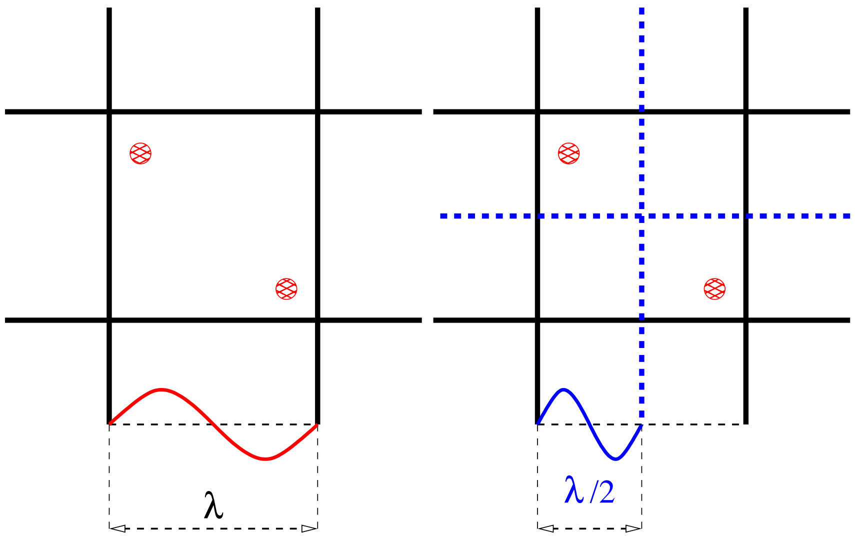
P.G., F. Picano, G. Sardina, C.M. Casciola, JFM **715** (2013)

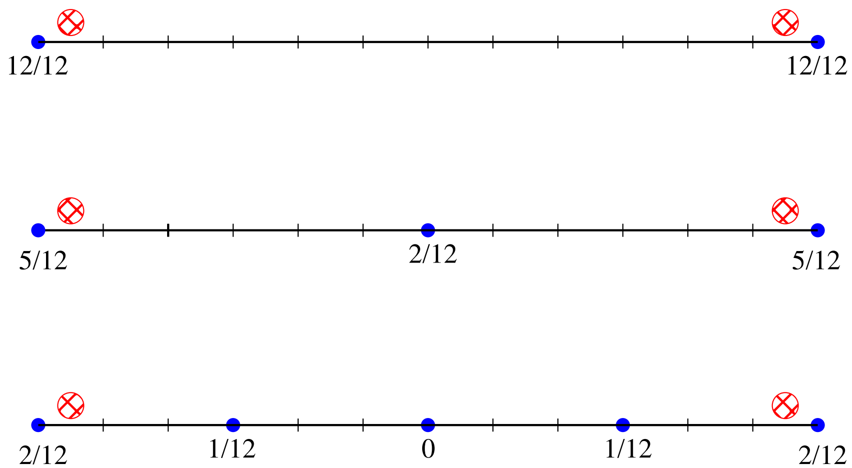
PIC: numerical issues

The *tails* *of the spectra* *hardly decay* *when* $N_p/N_c \ll 1$

- fine grids require a large number of particles
- grid dependent forcing
- limitations on the mass load

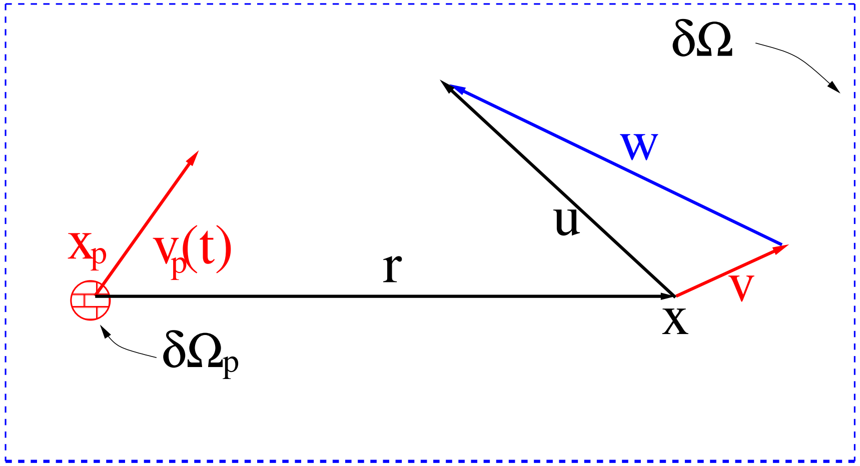






P.G., F. Picano, G. Sardina, C.M. Casciola, JFM **715** (2013)

Exact Regularized Point Particle (ERPP) method



$$\begin{cases} \dfrac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\nabla^2\mathbf{u} \\ \mathbf{u}|_{\partial\Omega_p} = \mathbf{v}_p; \quad \mathbf{u}|_{\partial\Omega} = \mathbf{u}_{wall} \\ \mathbf{u}(\mathbf{x}, t_n) = \mathbf{u}_0(\mathbf{x}) \end{cases}$$

Decompose the (incompressible) fluid velocity $\mathbf{u}$ in a background flow $\mathbf{w}$ and a perturbation $\mathbf{v}$, namely $\mathbf{u} = \mathbf{w} + \mathbf{v}$

$$\begin{cases} \dfrac{\partial \mathbf{w}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla \pi + \nu\nabla^2\mathbf{w} \\ \mathbf{w}|_{\partial\Omega} = \mathbf{u}_{wall} - \mathbf{v}|_{\partial\Omega} \\ \mathbf{w}(\mathbf{x}, t_n) = \mathbf{u}_0(\mathbf{x}) \end{cases}$$

$$\begin{cases} \dfrac{\partial \mathbf{v}}{\partial t} = -\nabla \tilde{p} + \nu\nabla^2\mathbf{v} \\ \mathbf{v}|_{\partial\Omega_p} = \mathbf{v}_p - \mathbf{w}|_{\partial\Omega_p} \\ \mathbf{v}(\mathbf{x}, t_n) = 0 \end{cases}$$

Perturbation $\mathbf{v}$ *described in terms of unsteady Stokes equations*

P.G., F. Picano, G. Sardina, C.M. Casciola, JFM **773** (2015)

ERPP: perturbation field

Exact solution of the unsteady Stokes problem

$$v_i(\mathbf{x},t) = \int_0^t d\tau \int_{\partial\Omega} t_j(\boldsymbol{\xi},\tau) G_{ij}(\mathbf{x},\boldsymbol{\xi},t,\tau) - v_j(\boldsymbol{\xi},\tau)\mathcal{T}_{ijk}(\mathbf{x},\boldsymbol{\xi},t,\tau) n_k(\boldsymbol{\xi})\, dS$$

G_{ij} the *unsteady Stokeslet*; $\mathcal{T}_{ijk}$ the associated stress tensor

For *small particles* the *far field* disturbance is estimated in terms of multipole expansion [Kim & Karilla, Microfluidics, (2000)]

$$v_i(\mathbf{x},t) \simeq -\int_0^t D_j(\tau) G_{ij}(\mathbf{x},\mathbf{x}_p,t,\tau)\, d\tau$$

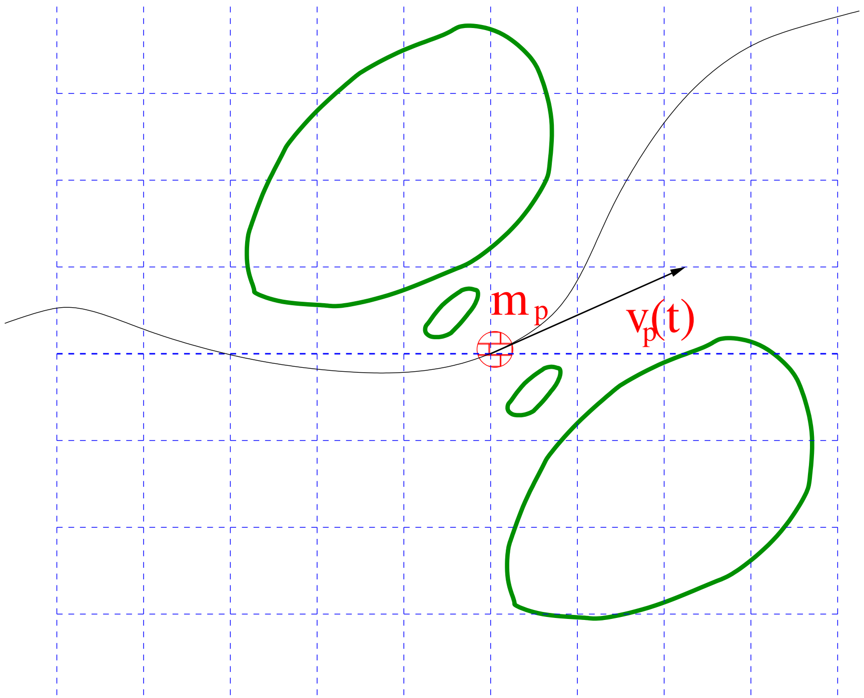
with $\mathbf{D}(\tau)$ hyrodynamic force, i.e. the Stokes Drag
($\cdots$ or more [Maxey & Riley, (1983); Gatignol (1983)])

ERPP: vorticity

Eulerian *far field* disturbance $\mathbf{v}(\mathbf{x}, t)$ described by the unsteady singularly forced Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} - \nu \nabla^2 \mathbf{v} + \nabla \tilde{p} = -\frac{\mathbf{D}(t)}{\rho_f} \delta \left[\mathbf{x} - \mathbf{x}_p(t) \right]$$

How to regularize the solution of the disturbance field?



Physics of the coupling

The vorticity, once generated along the particle trajectory, is diffused by viscosity and then injected into the Eulerian grid

vorticity generated by the particle ⇒ Eulerian Navier–Stokes solver

ERPP: vorticity diffusion

Why vorticity? $\Rightarrow$ Diffusion equation

$$\partial_t \zeta - \nu \nabla^2 \zeta = \frac{\mathbf{D}(t)}{\rho_f} \times \nabla \delta \left[\mathbf{x} - \mathbf{x}_p(t) \right]$$

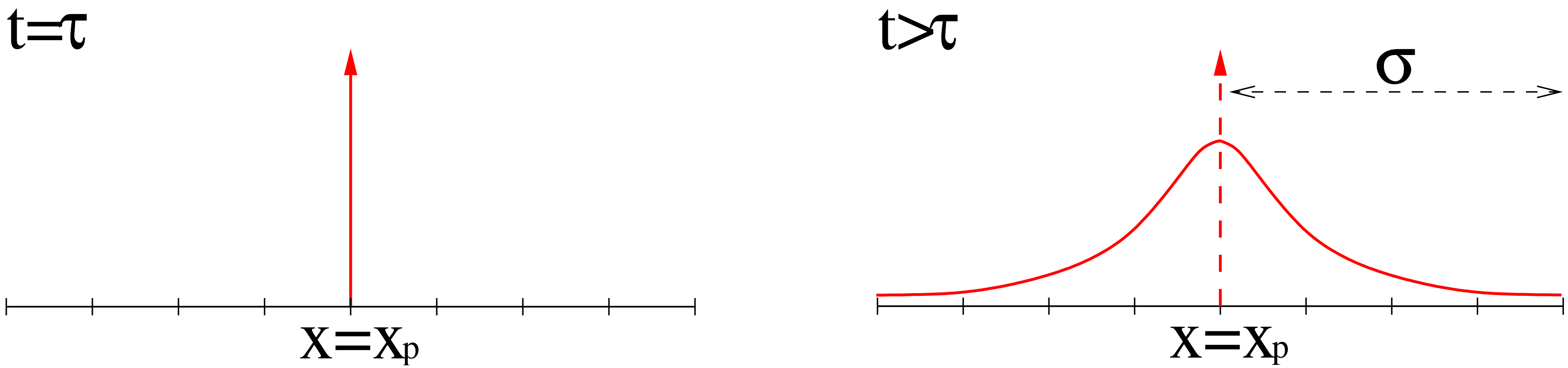
Fundamental solution

$$\partial_t g - \nu \nabla^2 g = \delta \left(\mathbf{x} - \mathbf{x}_p \right) \delta(t - \tau)$$

$$g(\mathbf{x}, \mathbf{x}_p, t, \tau) = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left(\frac{-\|\mathbf{x} - \mathbf{x}_p\|^2}{2\sigma^2} \right), \quad \sigma(t-\tau) = \sqrt{2\nu(t-\tau)}$$

For $t > \tau$ the solution is both

- *regular*, e.g. $g \in C^\infty$
- *local*, i.e decays more than exponentially



ERPP: vorticity regularization

- *Analytical solution* expressed as a convolution with the fundamental solution of the diffusion equation

$$\zeta(\mathbf{x},t) = \frac{1}{\rho_f}\int_0^t \mathbf{D}(\tau)\times\nabla g\left[\mathbf{x}-\mathbf{x}_p(\tau), t-\tau\right] d\tau$$

- For $\tau \simeq t$, $g(\mathbf{x},\mathbf{x}_p,t,\tau)$ tends to behave as badly as the Dirac delta function $\Rightarrow$ split $\zeta = \zeta_{Regular} + \zeta_{Singular}$

- The regularization procedure adopts a temporal cut-off ϵ_R

$$\zeta_R(\mathbf{x},t) = \frac{1}{\rho_f}\int_0^{t-\epsilon_R} \mathbf{D}(\tau)\times\nabla g\left[\mathbf{x}-\mathbf{x}_p(\tau), t-\tau\right] d\tau$$

$\Rightarrow$ Regularized field ζ_R everywhere smooth and characterized by the smallest spatial scale $\sigma_R = \sqrt{2\nu\epsilon_R}$

ERPP: coupling with the carrier phase

- The regular component of the vorticity field $\boldsymbol{\zeta}_R$ satisfy

$$\frac{\partial \boldsymbol{\zeta}_R}{\partial t} - \nu \nabla^2 \boldsymbol{\zeta}_R = \frac{1}{\rho_f} \nabla \times \mathbf{D}(t-\epsilon_R) g\left[\mathbf{x} - \mathbf{x}_p(t-\epsilon_R), \epsilon_R\right]$$

- The regular (perturbation) velocity field $\mathbf{v}_R$ follows as

$$\frac{\partial \mathbf{v}_R}{\partial t} - \nu \nabla^2 \mathbf{v}_R = -\frac{1}{\rho_f} \mathbf{D}(t-\epsilon_R) g\left[\mathbf{x} - \mathbf{x}_p(t-\epsilon_R), \epsilon_R\right]$$

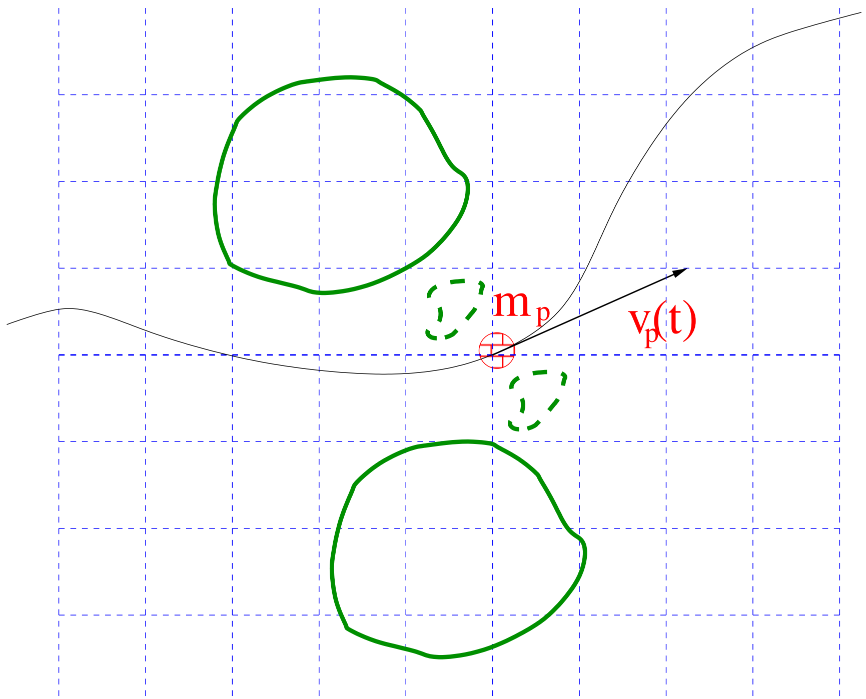
- The fluid velocity $\mathbf{u} = \mathbf{w} + \mathbf{v}_R$ is then given by

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u} - \sum_{p=1}^{N_p} \frac{\mathbf{D}_p(t-\epsilon_R)}{\rho_f} g\left[\mathbf{x} - \mathbf{x}_p(t-\epsilon_R), \epsilon_R\right]$$

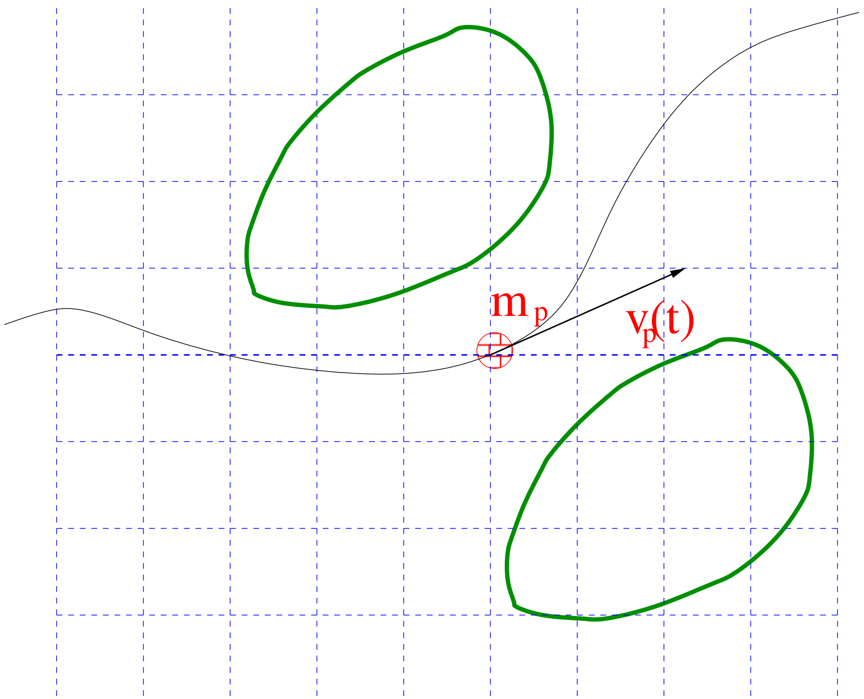
- Remarks
  - simply add an extra term in any $N-S$ solver
  - "anticipated" Green function: diffusion timescale ϵ_R
  - the function g is *local* in space $\Rightarrow$ *computational efficiency*

ERPP: a cartoon

complete field



regularized field

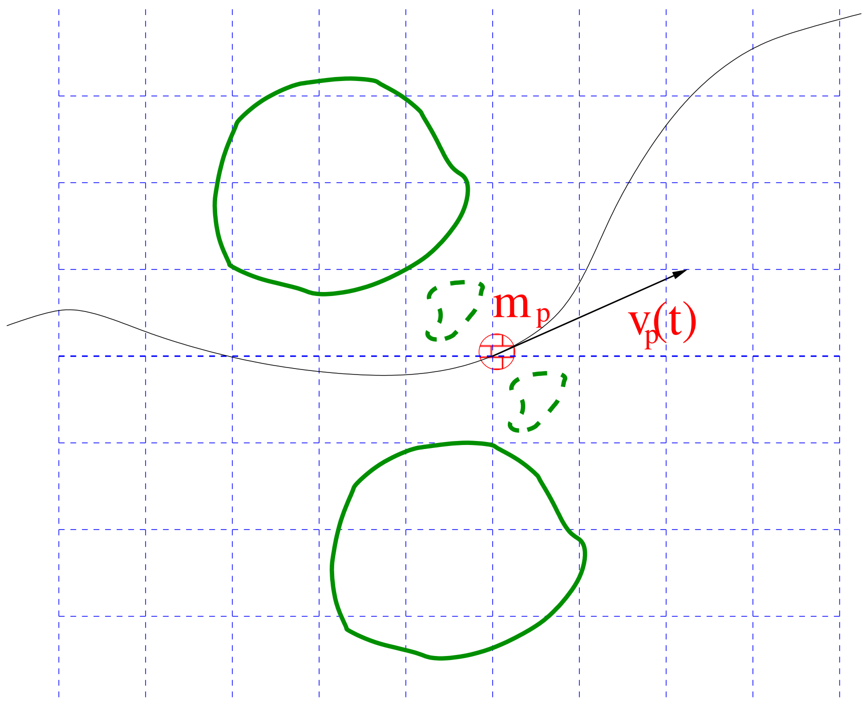


$$\boldsymbol{\zeta}(\mathbf{x}, t) = \frac{1}{\rho_f} \int_0^t \mathbf{D}(\tau) \times \nabla g \, d\tau \qquad \boldsymbol{\zeta}_R(\mathbf{x}, t) = \frac{1}{\rho_f} \int_0^{t-\epsilon_R} \mathbf{D}(\tau) \times \nabla g \, d\tau$$

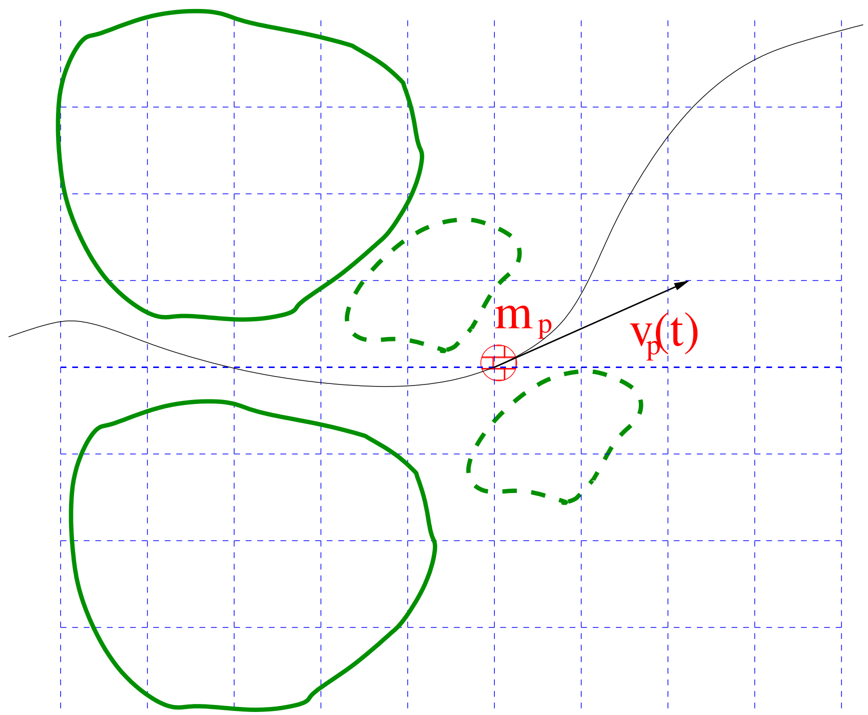
$\Rightarrow$ Vorticity at scales smaller that $\sigma_R = \sqrt{2\nu\epsilon_R}$ is not neglected but injected at later times

ERPP: a cartoon

complete field



regularized field at later times



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$\Rightarrow$ Vorticity at scales smaller that $\sigma_R = \sqrt{2\nu\epsilon_R}$ is not neglected but injected at later times

Hydrodynamic force in the two-way coupling regime

- Newton's law

$$m_p \frac{d\mathbf{v}_p}{dt} = \mathbf{D}_p(t) = 6\pi\mu a_p \left[\tilde{\mathbf{u}}(\mathbf{x}_p, t) - \mathbf{v}_p(t)\right]$$

$\Rightarrow$ $\tilde{\mathbf{u}}(\mathbf{x}_p, t)$ fluid velocity at $\mathbf{x}_p$ *in absence of the particle*

[Boivin et al. (1998); Jenny et al. (2012), P.G. et al. (2013,2015); Horwitz et al. (2016)]

- Removal of the self-disturbance $\mathbf{v}_{pth}$ from $\mathbf{u}(\mathbf{x}, t)$

$$\tilde{\mathbf{u}}(\mathbf{x}_p, t) = \mathbf{u}(\mathbf{x}_p, t) - \mathbf{v}_{pth}\left[\mathbf{x}_p(t) - \mathbf{x}_p(t_n); Dt\right]$$

- Self-disturbance velocity evaluated in closed form

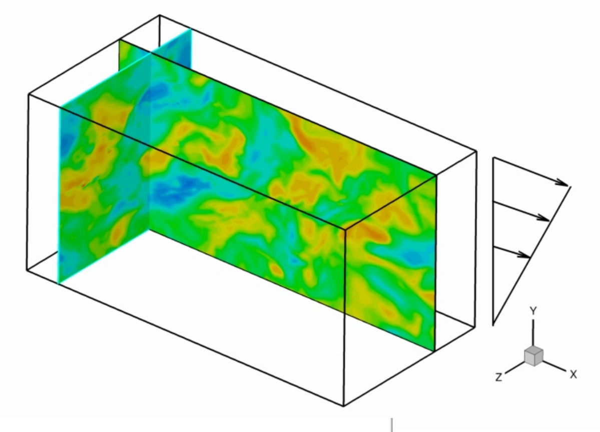
$$\mathbf{v}(\mathbf{r}, Dt) = \frac{1}{(2\pi\sigma^2)^{3/2}} \left\{ \left[e^{-\eta^2} - \frac{f(\eta)}{2\eta^3} \right] \mathbf{D}_p^n - \left(\mathbf{D}_p^n \cdot \hat{\mathbf{r}}\right) \left[e^{-\eta^2} - \frac{3f(\eta)}{2\eta^3} \right] \hat{\mathbf{r}} \right\}$$

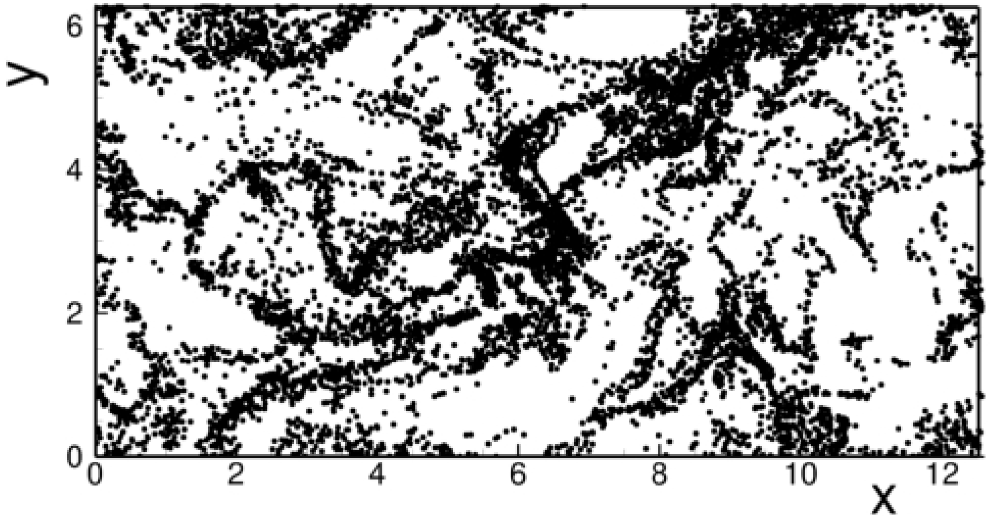
where $\mathbf{r} = \mathbf{x}(t) - \mathbf{x}_p(t_n)$; $\quad \eta = r/\sqrt{2}\sigma$; $\quad \sigma = \sqrt{2\nu(\epsilon_R + Dt)}$ and

$$f(\eta) = \frac{\sqrt{\pi}}{2}\mathrm{erf}(\eta) - \eta e^{-\eta^2}$$

Turbulent Flows: Homogeneos shear flow

$Re_\lambda = 80$
$St_\eta = 1,\ \Phi = 0.4$
$N_p = 2.200.000$
$d_p/\eta = 0.1$





- Remarks
  - regularization scale $\sigma_R = \eta$
  - the feedback field is everywhere *smooth*

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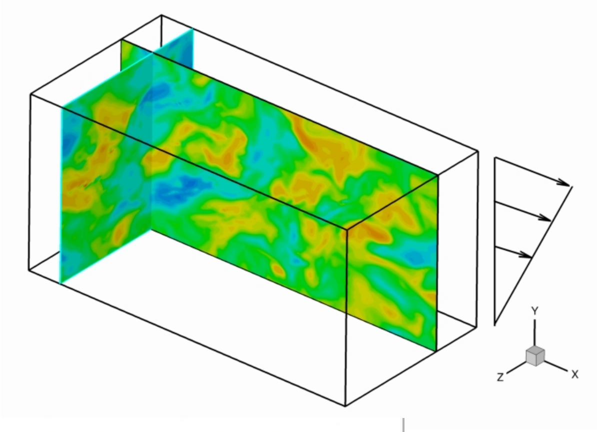

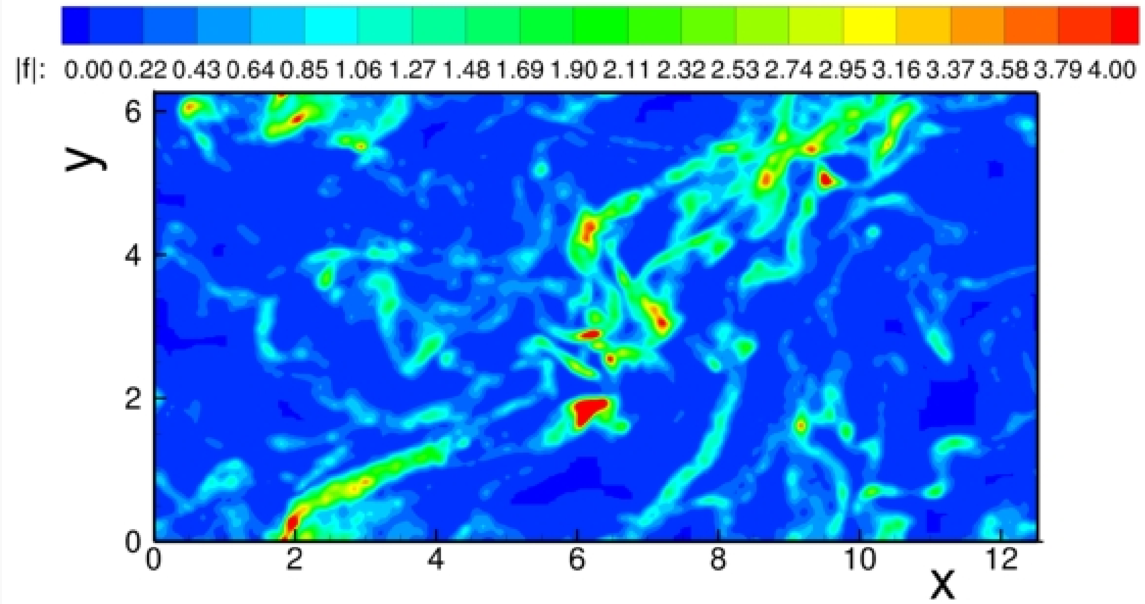

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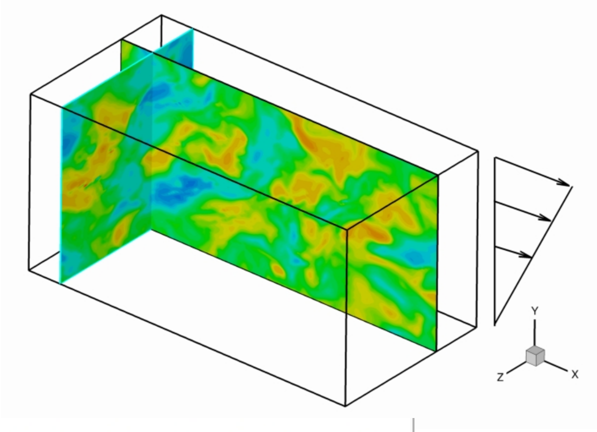

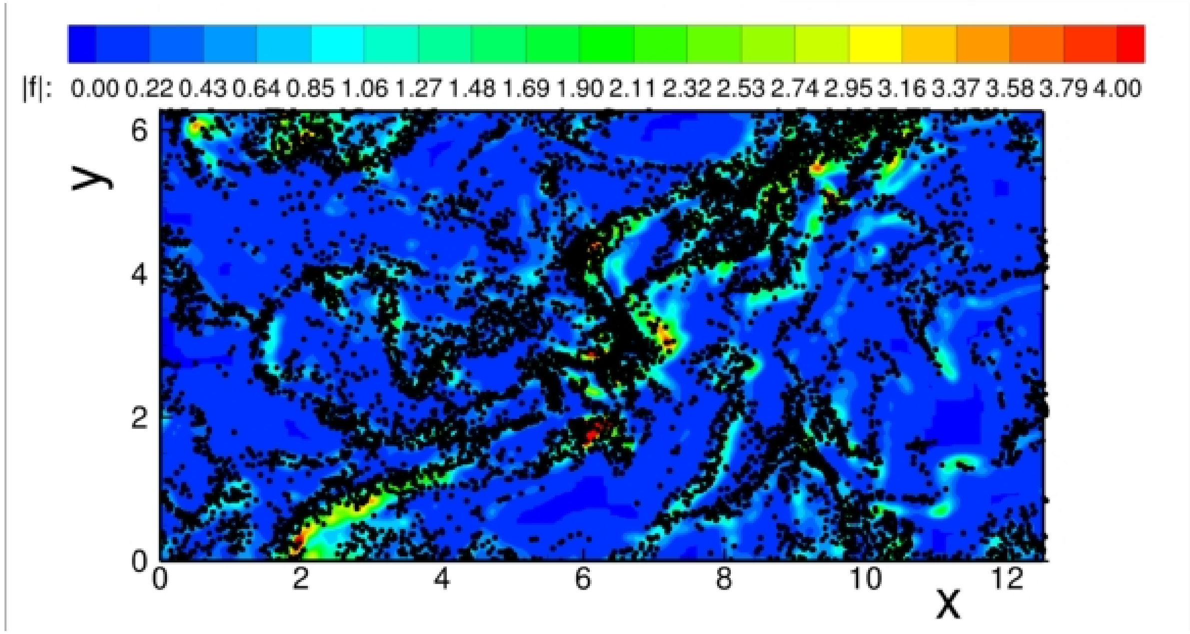

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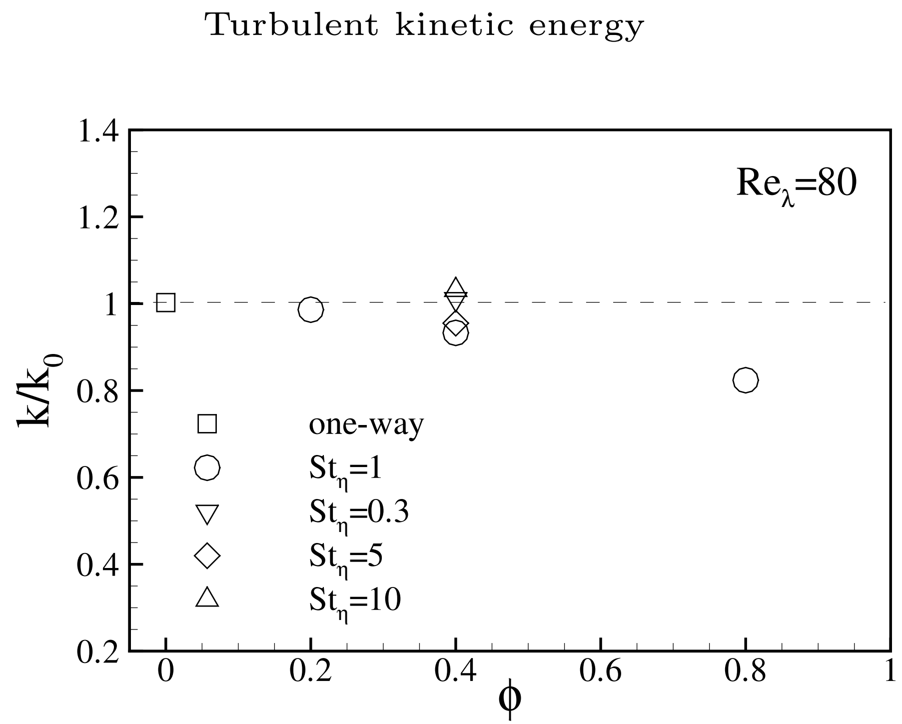
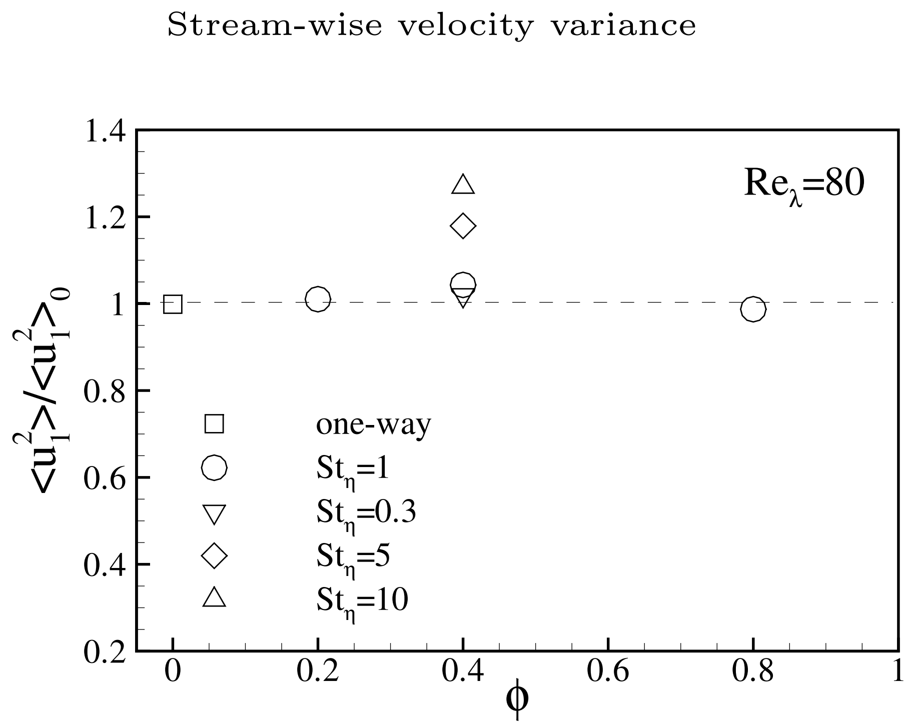
Turbulence modulation: velocity variances

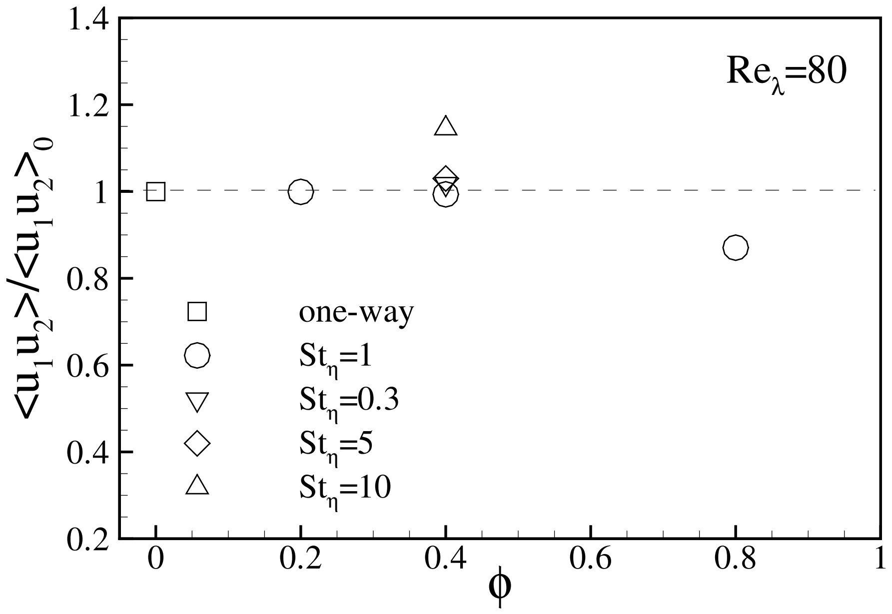




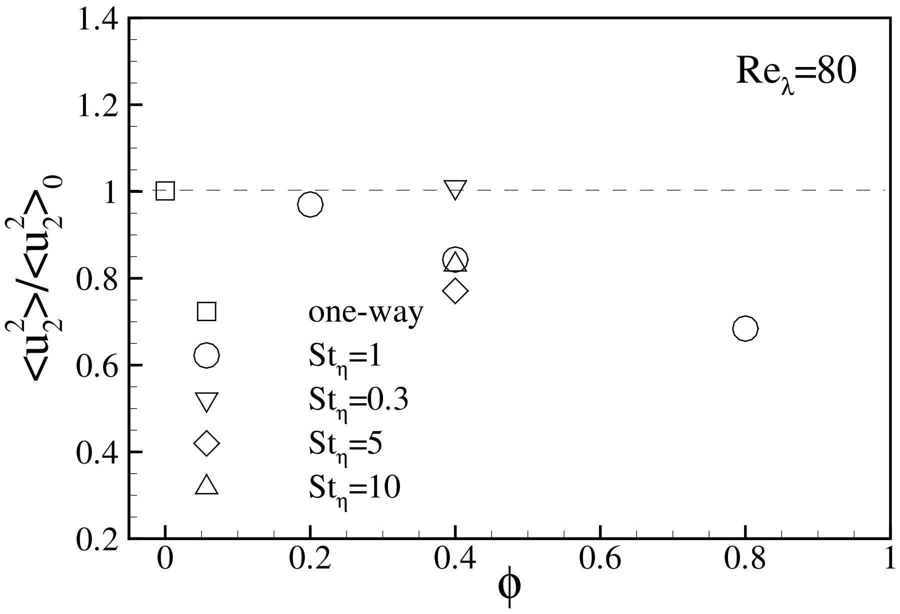
- Remarks
  - Fluctuations attenuated at increasing mass loadings
  - Selective turbulence modification
  - Effect of St_η on anisotropic turbulence modulation

Turbulence modulation: velocity variances

Reynolds shear stresses



Cross-flow velocity variance

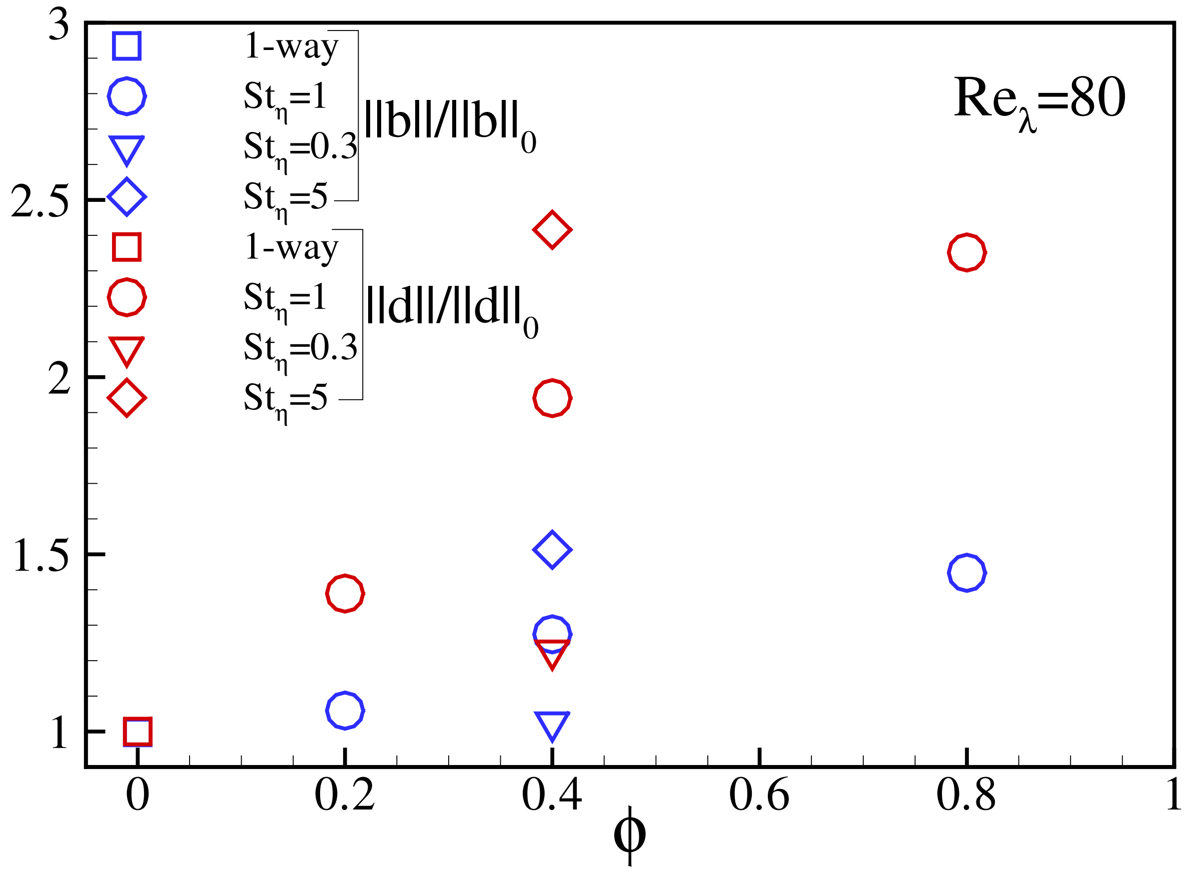


- Remarks
  - Fluctuations attenuated at increasing mass loadings
  - Selective turbulence modification
  - Effect of St_η on anisotropic turbulence modulation

Small scale isotropy recovery?

- Large & Small scale anisotropy indicator

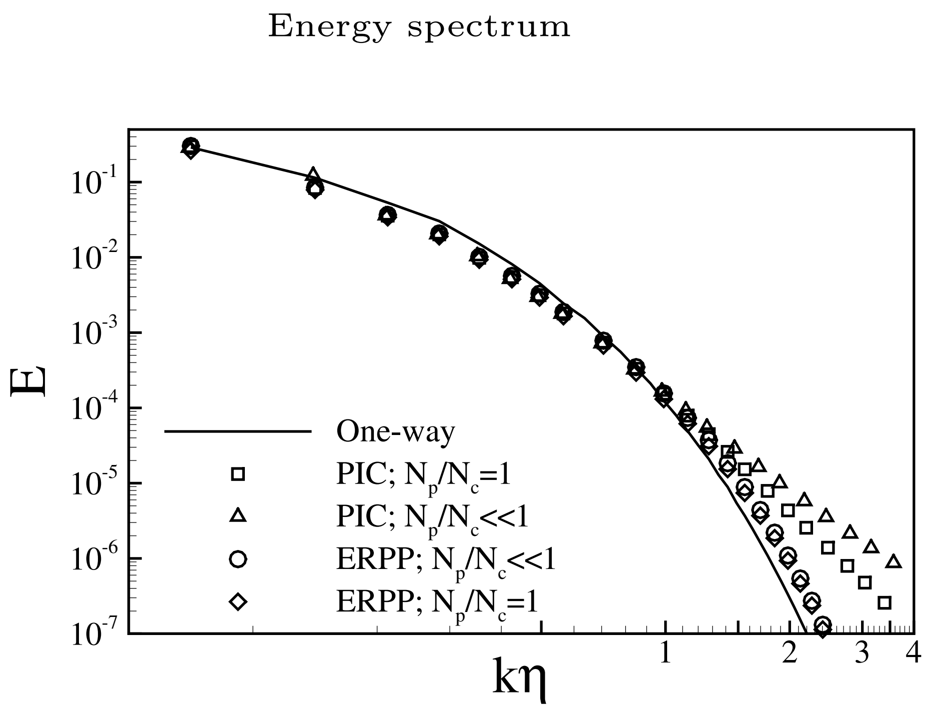
$$b_{\alpha\beta} = \frac{\langle u_\alpha u_\beta \rangle}{\langle u_\gamma u_\gamma \rangle} - \frac{1}{3}\delta_{\alpha\beta} \qquad d_{\alpha\beta} = \frac{\epsilon_{\alpha\beta}}{\epsilon_{\gamma\gamma}} - \frac{1}{3}\delta_{\alpha\beta}$$

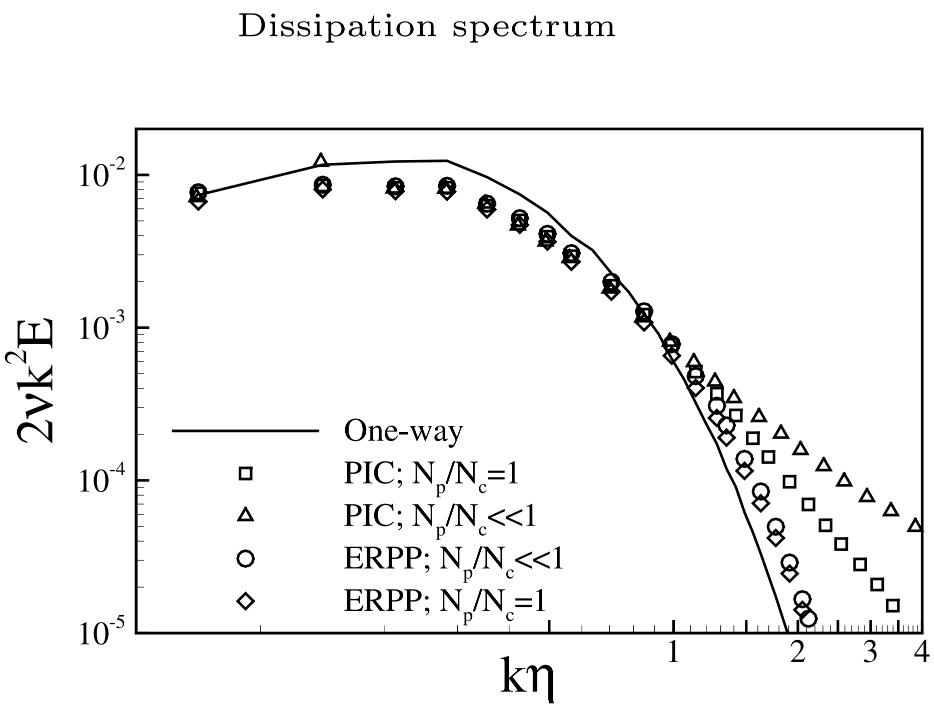


Anisotropy enhanced in the two-way coupling regime

Energy spectrum

- ERPP *vs.* PIC @ $\Phi = 0.4,\ St_\eta = 1$



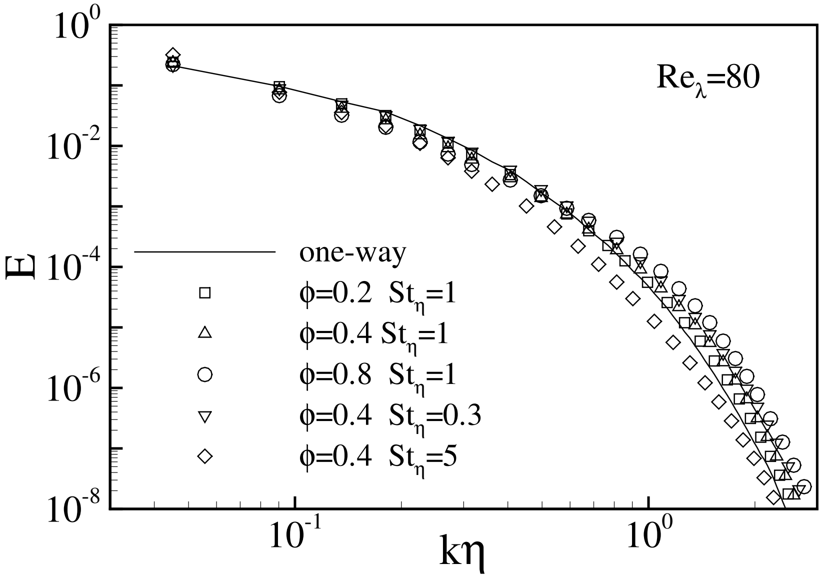


- Remarks
  - $E(k)$ and $2\nu\, k^2\, E(k)$ nicely smooth at small scales
  - ERPP prediction independent on N_p/N_c
  - Small scale fluctuations energized by the backreaction
  - Turbulence modulation controlled by Φ and St_η

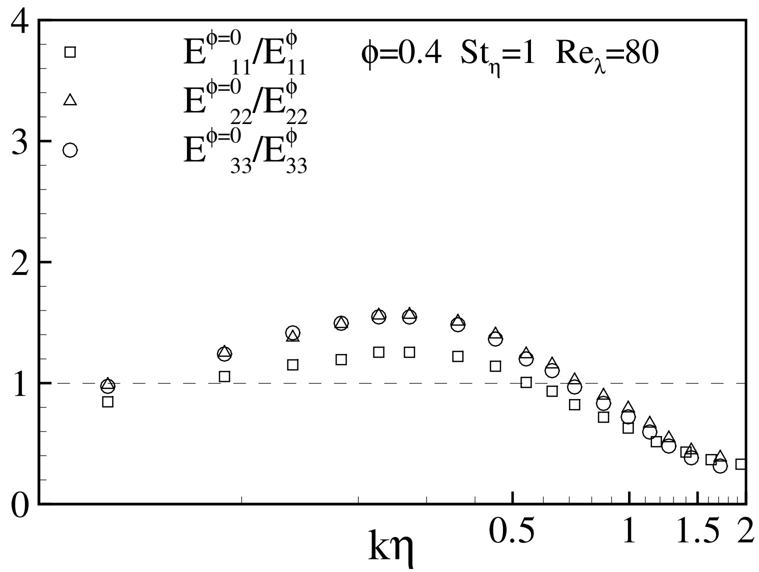
Turbulent spectra

- Effect of Mass loading Φ and Stokes number St_η

Energy spectrum



Velocity components spectrum

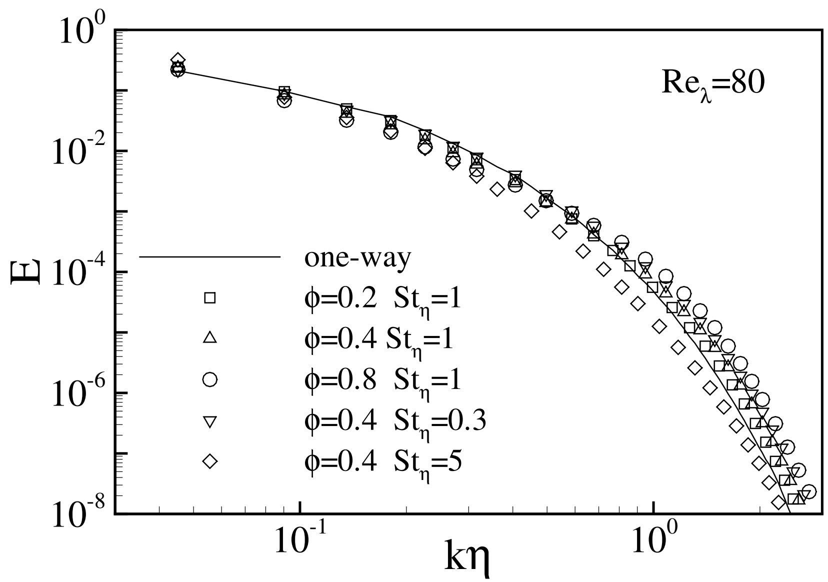


- Remarks
  - Small scale fluctuations energized at $St_\eta = 1$
  - Energy attenuation at all scales for $St_\eta = 5$
  - Anisotropic turbulence modulation at different scales

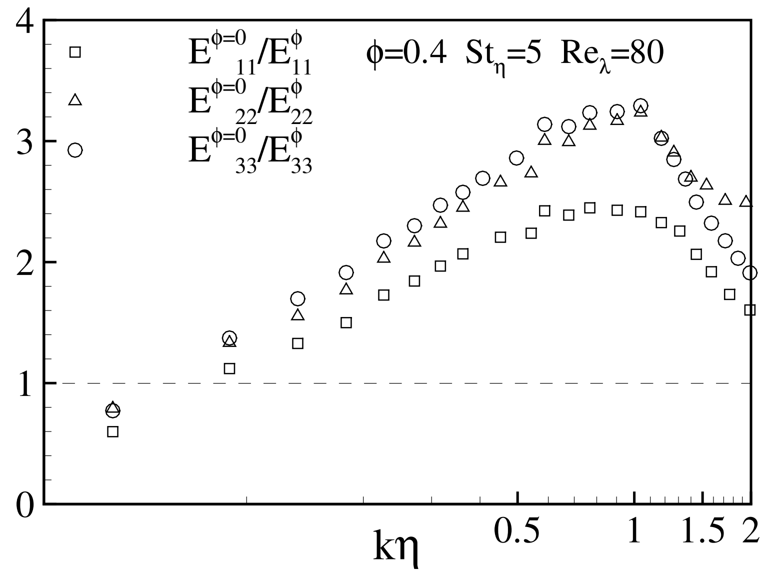
Turbulent spectra

- Effect of Mass loading Φ and Stokes number St_η

Energy spectrum



Velocity components spectrum

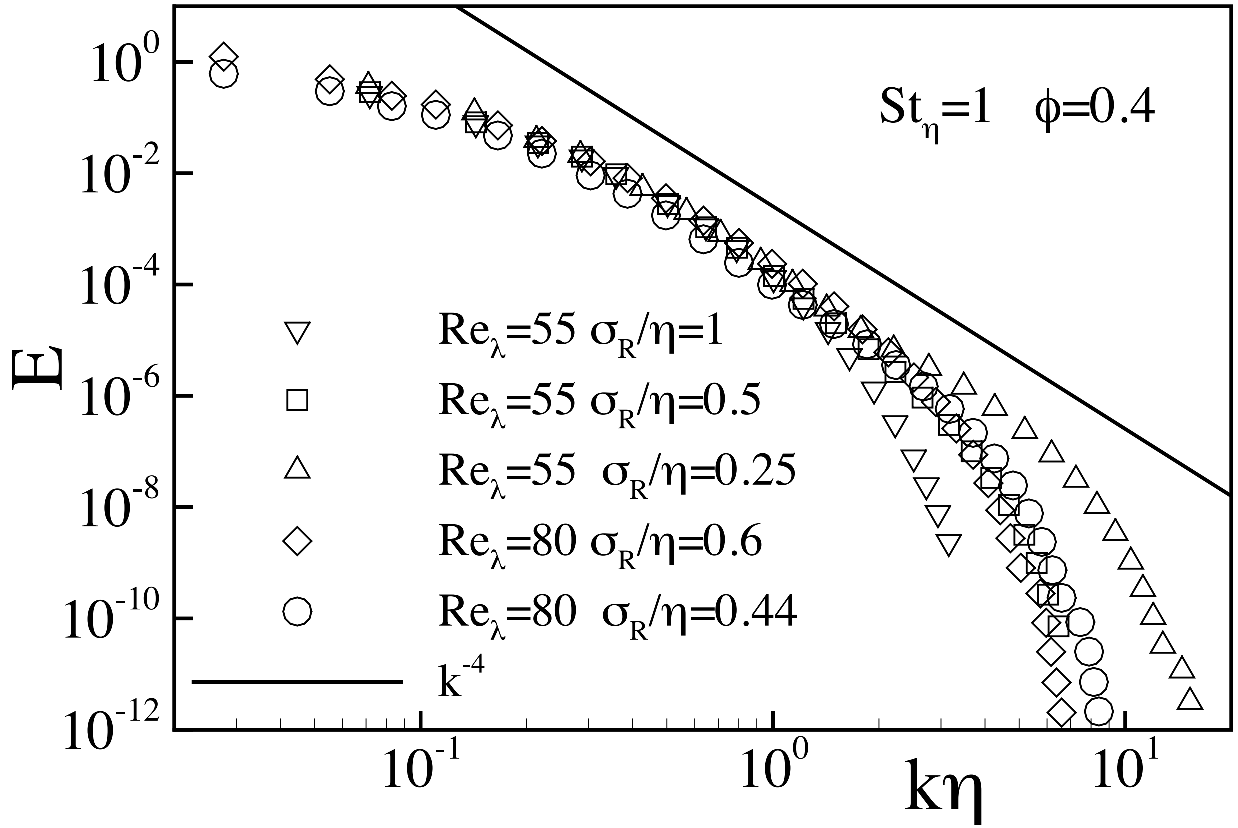


- Remarks
  - Small scale fluctuations energized at $St_\eta = 1$
  - Energy attenuation at all scales for $St_\eta = 5$
  - Anisotropic turbulence modulation at different scales

Energy spectrum: scaling in the two-way regime

- Enegy balance $T(k) + P(k) - D(k) + \Psi(k) = 0$
- At scales $k\eta \sim 1$ and $k\sigma_R < 1$

feedback ~ dissipation $\Rightarrow \Psi(k) \sim D(k) \Rightarrow E(k) \propto k^{-4}$



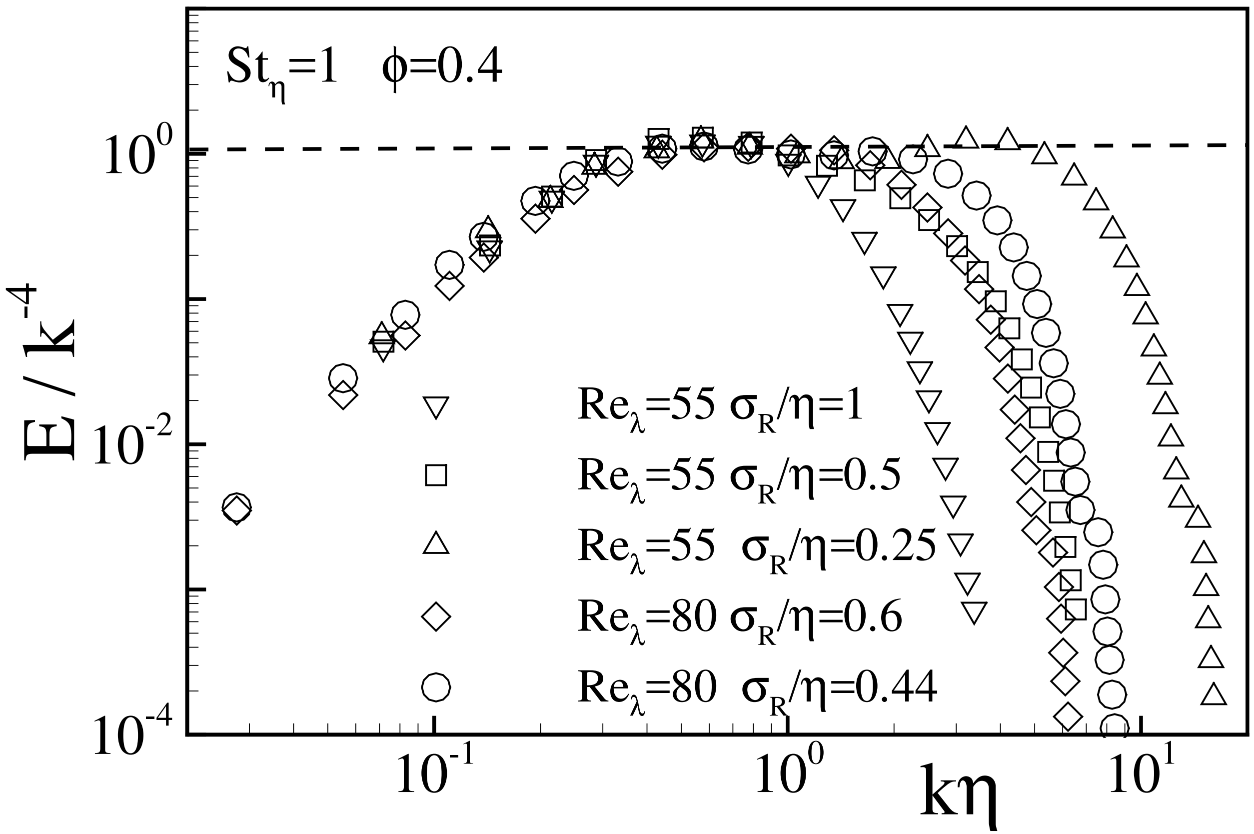
$\Rightarrow$ Convergent solutions w.r.t. regularization parameter σ_R/η

[P.G., Battista, C.M. Casciola, sub. Phys. Rev. Lett. 2016]

Energy spectrum: scaling in the two-way regime

- Enegy balance $T(k) + P(k) - D(k) + \Psi(k) = 0$
- At scales $k\eta \sim 1$ and $k\sigma_R < 1$

$$\text{feedback} \sim \text{dissipation} \Rightarrow \Psi(k) \sim D(k) \Rightarrow E(k) \propto k^{-4}$$



$\Rightarrow$ Convergent solutions w.r.t. regularization parameter σ_R/η

[P.G., Battista, C.M. Casciola, sub. Phys. Rev. Lett. 2016]

Karman-Howarth equation

- Scale-by-scale turbulence dynamics
  - production of scale energy $(L_0 < r < L_S)$
  - non linear energy transfer $(L_S < r < \eta)$
  - viscous effects $(r \sim \eta)$

$$\nabla_{\mathbf{r}} \cdot \mathbf{\Phi}_{\mathbf{r}} = \Pi_{\mathbf{r}} - 4\bar{\epsilon}$$

with

$$\mathbf{\Phi}_{\mathbf{r}} = \underbrace{\langle |\delta \mathbf{u}|^2 \, \delta \mathbf{u} \rangle}_{\text{inertial flux}} + \underbrace{2\nu \nabla_{\mathbf{r}} \langle |\delta \mathbf{u}|^2 \rangle}_{\text{viscous flux}} \qquad \Pi_{\mathbf{r}} = \underbrace{2\, S \langle \delta u \, \delta v \rangle}_{\text{production}}$$

⇒ In multiphase flows and extra term accounts for the *energy source/sink* due to the dispersed phase

$$\nabla_{\mathbf{r}} \cdot \mathbf{\Phi}_{\mathbf{r}} = \Pi_{\mathbf{r}} - 4\bar{\epsilon} + \Psi_{\mathbf{r}}$$

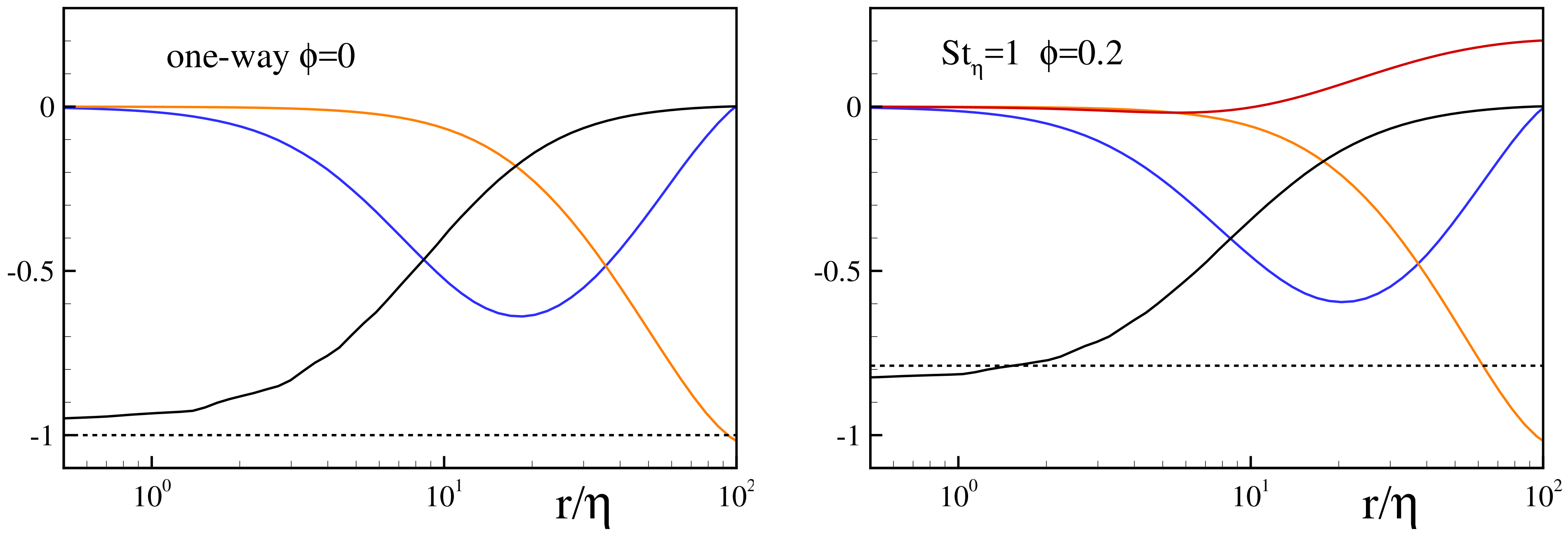
with

$$\Psi_{\mathbf{r}} = \underbrace{2 \langle \delta \mathbf{f} \cdot \delta \mathbf{u} \rangle}_{\text{feedback}}$$

KH budget in multiphase flows

- integrate over a ball $\mathcal{B}_r$ of radius r

$$S_3^{trn} + S_{uv}^{prd} + S_2^{dif} + S_{fu}^{prtcl} = -\frac{4}{3}\bar{\epsilon}r$$



- Effect of the particles
  - intercept energy at large scale and release it at small scales
  - stall the energy cascade
  - small-scale pumping

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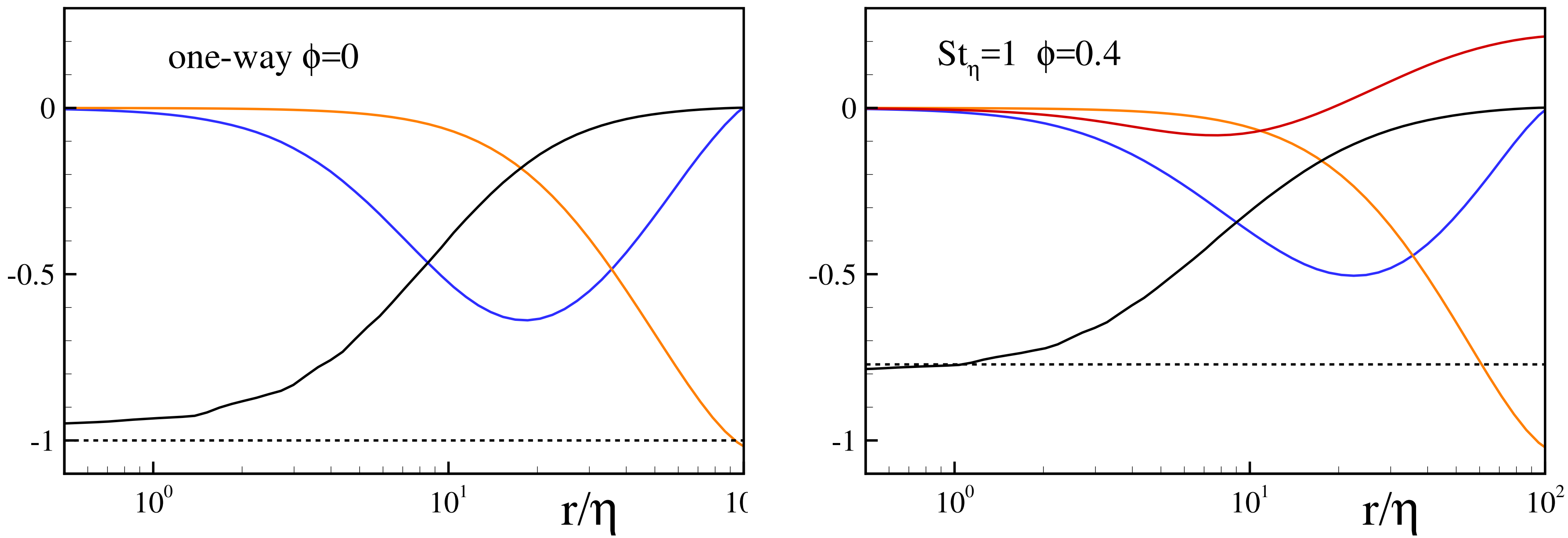

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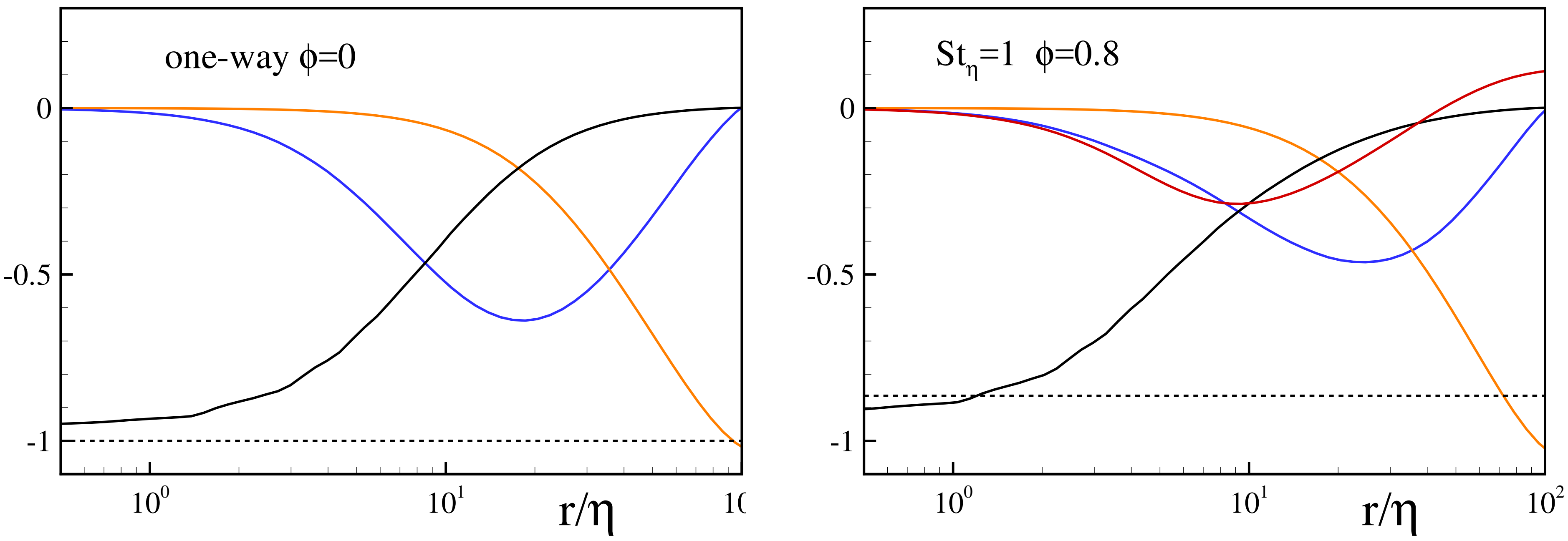

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Final remarks

- ERPP method
  - based on physical arguments $\Rightarrow$ Unsteady Stokes Solutions
  - easy to implement and portable
- Turbulence in multiphase flows
  - selective modification of turbulence intensities
  - scaling $E(k) \propto k^{-4}$ where feedback $\sim$ dissipation
  - energy cascade stalled at high mass loadings
  - (anisotropic) pumping at small scales

□ **Credits**

- PRACE projects FP7 RI-283493 and #2014112647
- ERC grant #339446 **BIC: Bubbles from Inception to Collapse**, P.I. *C.M. Casciola*