Introduction to (Large) Extra Dimensions

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Outline

- Introduction
- Small vs Large Extra Dimensions (LED)
- Braneworld
- Kaluza-Klein (KK) Particles
- Orbifolds
- Low Energy Theory and Phenomenology
- Cosmology & Astrophysics
- Extra Dimensions and GUT

Introduction

General framework: class of models with n extra space-time (usually space-like) dimensions

$$D = 4 + n$$

First attempt ($\sim$ 1920) $\Rightarrow$ Kaluza and Klein

Unification of the electromagnetism with the Einstein gravity with a compact 5^{th} dimension.

The photon comes from the extra components of the metric.

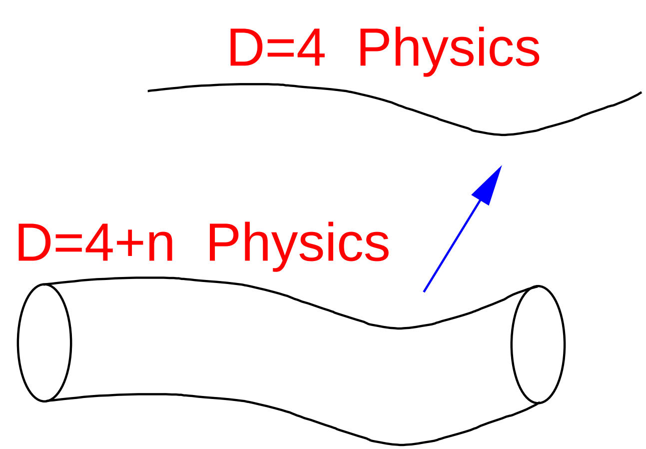
Small Extra Dimensions

Early assumption: extra dimensions are "small" and are compactified to manifolds (typical $S^1 \times \cdots \times S^1$) of small "radii" of the order of the Planck length

$$l_\mathrm{P} = \left(\frac{8\pi G_\mathrm{N}\hbar}{c^3}\right)^{1/2} \sim 10^{-33}\,\mathrm{cm}$$

related to the Planck mass (the relevant quantum gravity/string scale)

$$M_\mathrm{P} = \left(\frac{\hbar c}{8\pi G_N}\right)^{1/2} \sim 2.4\times 10^{18}\,\mathrm{GeV}$$



Large Extra Dimensions (ADD Model)

Gravity coupling in higher dimensional model G_*. In $D = 4$ $G_* = G_N$, the Newton constant.

Let us assume a toroidal compactification of radius R ($M^4 \times T^n$). The higher dimensional Einstein-Hilbert action is

$$S_{grav} = \frac{1}{16\pi G_*} \int d^{4+n}x \sqrt{|g_{(4+n)}|} R_{(4+n)}$$

where $g_{(4+n)}$ is the $D = 4 + n$ metric

$$ds^2 = g_{MN} dx^M dx^N$$

with $M, N = 0, 1, \cdots, 3 + n$

Dimensional Analysis

Assuming $g_{(4+n)}$ dimensionless.

- $\left[R_{(4+n)}\right] = [\mathrm{Length}]^{-2} = [\mathrm{Mass}]^{2}$
- $[G_N] = [\mathrm{Mass}]^{-2}$
- $[G_*] = [\mathrm{Mass}]^{-(2+n)}$

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The dimension of a scalar field ϕ is obtained from the kinetic term $\int d^4x d^n y \partial^A \phi \partial_A \phi$ that implies

- $[\phi] = [\mathrm{Mass}]^{1+n/2}$

Dimensional Reduction

In order to obtain the $D = 4$ effective action let us assume a flat ansatz (vs warped ansatz)

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu - \delta_{ab}dy^a dy^b$$

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$$\Downarrow$$

$$\boxed{\sqrt{|g_{(4+n)}|} = \sqrt{|g_{(4)}|} \qquad R_{(4+n)} = R_{(4)}}$$

Effective Action

The effective $D = 4$ action is

$$S_{grav} = \frac{V_n}{16\pi G_*} \int d^4x \sqrt{\left|g_{(4)}\right|} R_{(4)}$$

where V_n is the volume of the extra space.

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The action S_{grav} is precisely the standard gravity action upon the identification

$$G_N = \frac{G_*}{V_n}$$

Newton Law in Higher Dimensions

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$$U(r) = -G_N \frac{m_1 m_2}{r}$$

- $r \ll R \Rightarrow$ the $D = 4$ observer is able to feel the presence of the bulk

$$U_*(r) = -G_* \frac{m_1 m_2}{r^{1+n}}$$

Planck Scale

In higher dimensional theory the fundamental (quantum) gravity scale is given in terms of G_*

$$M_* c^2 = \left(\frac{\hbar^{1+n} c^{5+n}}{8\pi G_*} \right)^{1/(2+n)}$$

Turning to natural units we can see the relation between the $D = 4$ and the $D = 4 + n$ Planck scale

$$\boxed{M_P^2 = M_*^{2+n} V_n}$$

Fundamental Interactions

From particle physics there is no evidence of quantum gravity, supersymmetry or string effects up to energies around few hundred GeV

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No hierarchy in the fundamental scale of physics

but now we have to explain why the extra dimensions are large!

Size of the Compact Dimensions

Assuming a toroidal compactification with all the S^1 of the same size R and $M_* \sim m_{EW}$ in the relation (*)

$$R \sim 10^{30/n-17}\,\mathrm{cm} \times \left(\frac{1\,\mathrm{TeV}}{m_{EW}}\right)^{1+2/n}$$

n	R
1	10^{11} m
2	0.2 mm
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current limit of short distance gravity experiments

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Usually at fixed points (*) of the compact space

$$\Downarrow$$

breaking of translational invariance

Braneworld

Two kind of fields: bulk and brane (boundary) fields

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- bulk theory: they describe gravity and other fields that can propagate in the bulk $\phi(x^\mu, \vec{y})$

$$S_{bulk}[\phi] = \int d^4x d^n y \sqrt{\left|g_{(4+n)}\right|} \mathcal{L}\left(\phi(x^\mu, \vec{y})\right)$$

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- bulk brane coupling terms:

$$\int d^4x d^n y \sqrt{\left|g_{(4)}\right|} \phi(x^\mu, \vec{y}) \bar{\psi}(x^\mu) \psi(x^\mu)\, \delta^n\left(\vec{y} - \vec{y}_0\right)$$

Kaluza Klein Modes

Effective $D = 4$ theory coming from a $D = 5$ theory compactified on a circle of radius R.

The higher dimensional action (in flat space-time) for a massive bulk scalar field ϕ is

$$S[\phi] = \frac{1}{2} \int d^4x dy \left(\partial^A \phi \partial_A \phi - m^2 \phi^2 \right)$$

where $A = 1, \cdots, 5$.

The compactness of the internal manifold is reflected in the periodicity of the field

$$\phi(y) = \phi(y + 2\pi R)$$

Kaluza Klein Modes

The periodicity condition allows a Fourier decomposition

$$\phi(x,y) = \frac{1}{\sqrt{2\pi R}} \quad \phi_0(x) \quad + \sum_{k=1}^{\infty} \frac{1}{\sqrt{\pi R}} \left[\phi_k(x) cos\left(\frac{ky}{R}\right) + \hat{\phi}_k(x) sin\left(\frac{ky}{R}\right) \right]$$

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- the 0-mode has no dependence from $5^{\rm th}$ dimension
- the fields $\phi_k(x)$ and $\hat{\phi}_k(x)$ are called the excited or Kaluza-Klein (KK) modes

There is a different normalization on all the excited modes

The expansion can be also performed using $e^{iny/R}$ (*)

KK spectrum

Inserting back the Fourier expansion into $D=5$ action and integrating over the extra dimensions we obtain

$$S[\phi] = \sum_{k=0}^{\infty} \frac{1}{2} \int d^4x \left(\partial^\mu \phi_k \partial_\mu \phi_k - m_k^2 \phi_k^2\right) + + \sum_{k=1}^{\infty} \frac{1}{2} \int d^4x \left(\partial^\mu \hat{\phi}_k \partial_\mu \hat{\phi}_k - m_k^2 \hat{\phi}_k^2\right)$$

where the KK mass is given by $m_k^2 = m^2 + k^2/R^2$

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\begin{aligned}
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&\quad + \sum_{k=1}^{\infty} \frac{1}{2} \int d^4x \left(\partial^\mu \hat{\phi}_k \partial_\mu \hat{\phi}_k - m_k^2 \hat{\phi}_k^2\right)
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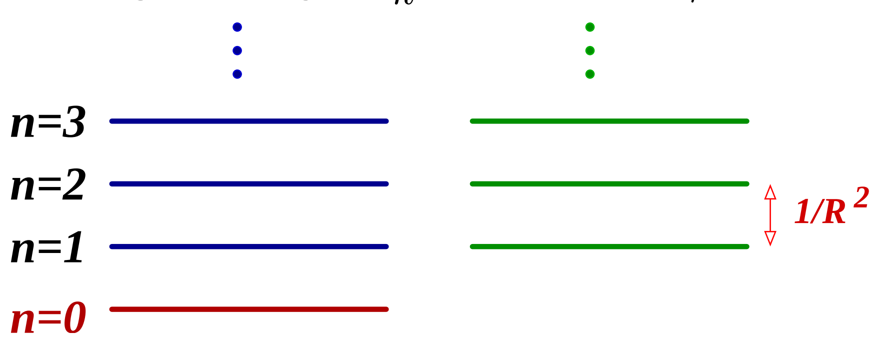
where the KK mass is given by $m_k^2 = m^2 + k^2/R^2$



KK spectrum

The KK number is associated to the 5^{th} component of the momentum.
In terms of higher dimensional invariant

$$p^A p_A = m^2$$

that can be rewritten in terms of effective $D = 4$ momentum

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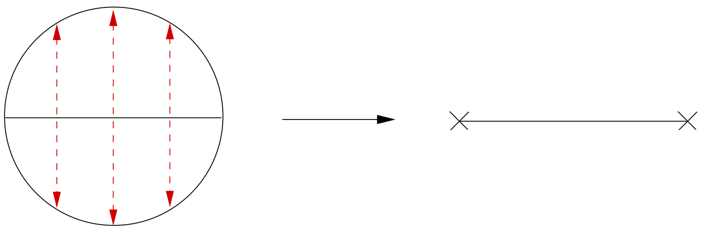
Different compactifications lead to
different mode expansion

Orbifolds

Extra boundary conditions on the compact space. Orbifold example

$$S^1/\mathbb{Z}_2$$

which is built out of the circle identifying opposite points



the theory has to be invariant under the extra parity symmetry $\mathbb{Z}_2 : y \to -y$. All the fields pick up a specific parity

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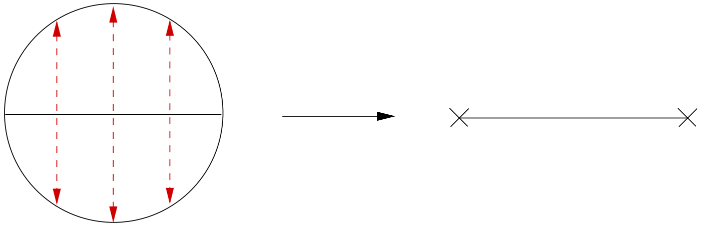

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The theory has only half of the KK-modes

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- for energies $E \ll 1/R$ physics behave purely as in $D = 4$
- at larger energies $1/R < E < M_*$ (shorter distances) a larger number of KK excitations $\sim (ER)^n$ becomes kinematically accessible
- for $E > M_*$ the effective approach has to be replaced by the use of a fundamental theory (string theory)

Coupling Suppressions

Consider a bulk scalar field $\phi(x,y)$ in $D=5$. Due to the change in the dimension of the field the interaction terms in the lagrangian (apart for the mass term) have dimensionful couplings.

For example the quartic coupling

$$\frac{\lambda}{M_*}\phi^4$$

with λ dimensionless.

The effective coupling for the $D=4$ KK-modes is given by

$$\lambda\left(\frac{M_*}{M_P}\right)^2\phi_k\phi_l\phi_m\phi_{k+l+m}$$

the indices structure reflects the momentum conservation along y

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Recall that the same happens to gravity in the bulk.

Brane-Bulk Couplings

Consider a brane fermion field $\psi(x)$ coupled to a bulk scalar field $\phi(x, y)$. Let us assume the brane located in $y = 0$ (one of the orbifold fixed point).

The action that describes this coupling is

$$\int d^4x dy \frac{h}{\sqrt{M_*}} h\bar{\psi}(x)\psi(x)\phi(x, y = 0)\delta(y) \quad = \int d^4x \frac{M_*}{M_P} h\bar{\psi}\psi \left(\phi_0 + \sqrt{2}\sum_{k=1}^{\infty}\phi_k\right)$$

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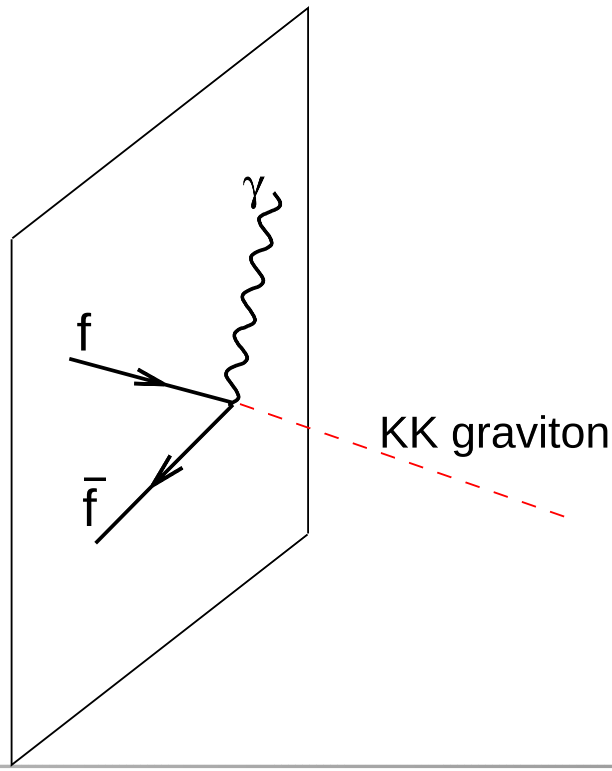
note that the odd modes $\hat{\phi}_k$ decouple from the brane

Missing Energy Processes

Brane-Bulk couplings in general do not conserve the KK number because branes break the translational symmetry along the extra dimensions. For example (tree level) KK production:



$D = 4$ theory is still Lorentz invariant

Part of the brane energy is released into the bulk

KK Graviton Production

Much of the brane energy could go into the bulk through KK graviton production. The splitting among those KK-modes

$$\Delta m_{KK} = R^{-1} = M_* \left(\frac{M_*}{M_P}\right)^{2/n} = \left(\frac{M_*}{\mathrm{TeV}}\right)^{1+n/2} 10^{\frac{12n-31}{n}}\ \mathrm{eV}$$

For $n = 2$ and $M_* \sim 1$ TeV $\Rightarrow \Delta m_{KK} \simeq 10^{-3}$ eV. The number of KK modes kinematically accessible (E center of mass energy)

$$N = (ER)^n$$

The graviton creation rate at temperature T, per unit time and volume

$$\sigma_{KK\ graviton} = \frac{(TR)^n}{M_P^2} = \frac{T^n}{M_*^{n+2}}$$

Cosmological Bounds

During BBN $E \simeq 1$ MeV. This implies for $n = 2$ there are $N \simeq 10^{18}$ KK modes available. The standard Universe evolution still holds if

$$\frac{n_{KKgraviton}}{n_\gamma} \simeq \frac{T^{n+1} M_P}{M_*^{n+2}} < 1$$

This implies a bound for the maximal reheating temperature our Universe can reach

$$T_r^{n+1} < \frac{M_*^{n+2}}{M_P}$$

for example $M_* \simeq 10$ TeV and $n = 2$ imply $T_r < 100$ MeV

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braneworlds predict a cold Universe

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Thermal graviton emission happens in many astrophysical objects

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- SN cooling (data from SN1987a)

$$M_* \gtrsim 10^{\frac{15-4.5n}{n+2}}$$

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- Diffuse γ-ray background ($g_{KK} \to \gamma\gamma, e^+e^-$). The mean lifetime for a KK graviton is

$$\tau_{g_{KK}} \simeq 10^{11}\,\text{years} \cdot (30\,\text{MeV}/m_{g_{KK}})^3$$

  for $m_{g_{KK}} \simeq 30$ MeV they decay at the present time and this implies (from EGRET and COMPTEL) $\Rightarrow M_* > 500\,\text{TeV}$

Astrophysical Bounds

Stringent limits come from the observation of neutron stars

Massive KK gravitons have small kinetic energy so that a large fraction of those produced in the inner SN core remain gravitationally trapped

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Neutron stars dark except for $g_{KK} \to \gamma\gamma,\ e^+e^-$

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GLAST will be able to test models with a fundamental scale as large as $M_* = 1300$ TeV for $n = 2$

Power Law Running

The presence of KK towers has dramatic effects on the RGE of the MSSM

$$\frac{d\alpha_i^{-1}}{dt} = -\frac{b_i}{2\pi}$$

where $t = ln(\mu)$ and $\alpha_i = g_i^2/4\pi$, $i = 1, 2, 3$ are the MSSM coupling constants.

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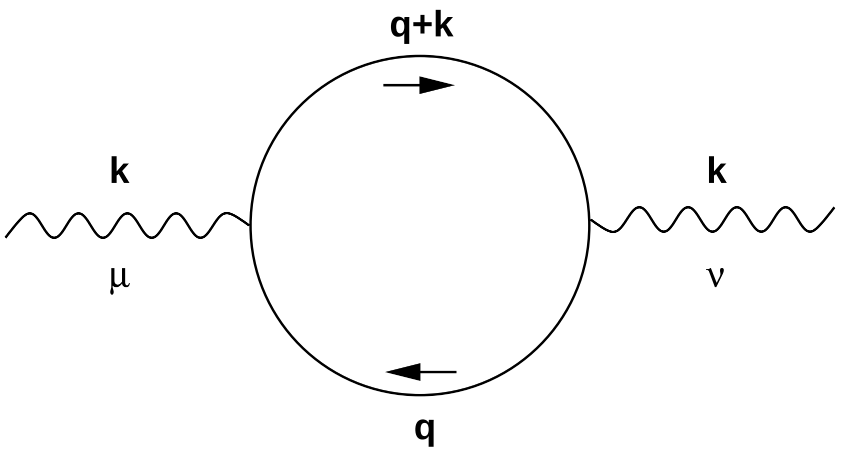
The effects on the MSSM gauge couplings can be obtained computing



where in the loop you have to consider the contribution of every KK mode

Power Law Running

The result (after truncation of the effective theory) is that the running couplings acquire a power law behaviour

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{b_i - \tilde{b}_i}{2\pi} ln\left(\frac{\Lambda}{\mu_0}\right) - \frac{\tilde{b}_i X_\delta}{2\pi\delta}\left[\left(\frac{\Lambda}{\mu_0}\right)^\delta - 1\right]$$

where

$$X_\delta = \frac{\pi^{\delta/2}}{\Gamma(1+\delta/2)} = \frac{2\pi^{\delta/2}}{\delta\Gamma(\delta/2)}$$

is the volume of the δ-dim unit sphere.

Power Law Running

The result (after truncation of the effective theory) is that the running couplings acquire a power law behaviour

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{b_i - \tilde{b}_i}{2\pi} ln\left(\frac{\Lambda}{\mu_0}\right) - \frac{\tilde{b}_i X_\delta}{2\pi\delta}\left[\left(\frac{\Lambda}{\mu_0}\right)^\delta - 1\right]$$

where

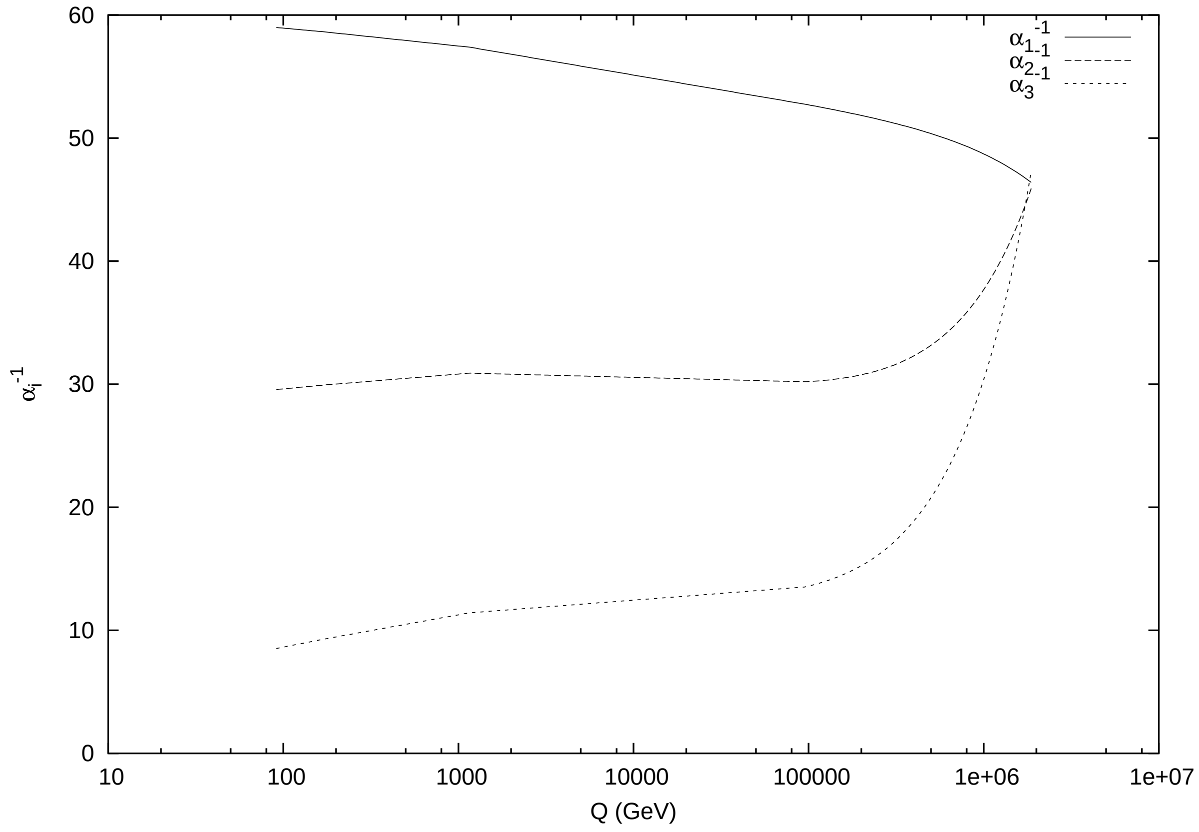
$$X_\delta = \frac{\pi^{\delta/2}}{\Gamma(1+\delta/2)} = \frac{2\pi^{\delta/2}}{\delta\Gamma(\delta/2)}$$

is the volume of the δ-dim unit sphere.

the $\tilde{b}_i$ are the β-function coefficients of the KK mode

Power Law Unification

Power law running $\Rightarrow$ GUT scale much lower than $\simeq 10^{16}$ GeV.



Extra dimensions effects for $Q > \mu_0$. In this case $\mu_0 \simeq 10^5$ GeV and $M_{GUT} \simeq M_* = 10^5$ GeV

Soft Masses

All the soft parameters receive power law corrections

$$\left(\frac{\Lambda}{\mu_0}\right)^{\delta}$$

with the proportionality coefficients different in the case you have $\eta = 1, 2, 3$ fermion generations that propagates in the bulk, *i.e.* they possess KK towers.

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Once you compute (now completed!) all the KK corrections it is possible to derive all the phenomenological consequences:

- spectrum of the MSSM fields (neutralino LSP??)
- cross sections
- flux and signals coming from the LSP

Conclusions

- ED consitute a very general framework with new concepts and lots of possibilities
- There are consistent models with LED
- Many phenomenological consequences with distinctive experimental signatures
- Require thinking about how to find them
- Other scenarios can be explored (Warped ED, UED)

Bulk and Brane Fields

Example in $D = 5$, $n = 1$ extra space-like coordinate

- Bulk fields: any interaction involving these fields conserve the KK number

$$\Phi\left(x^{\mu}, y\right) = \sum_{k=-\infty}^{\infty} \phi_k\left(x^{\mu}\right) e^{iky/R}$$

with $\phi_{-k} = \phi_k^*$ Even and odd fields under $\mathbb{Z}_2$

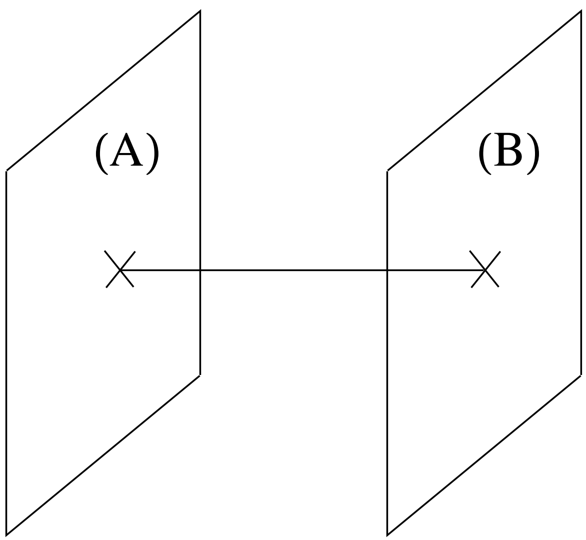
$$\Phi_+\left(x^{\mu}, y\right) = \sum_{k=0}^{\infty} \left[\phi_k\left(x^{\mu}\right) + \phi_{-k}\left(x^{\mu}\right)\right] cos\left(ky/R\right)$$

$$\Phi_-\left(x^{\mu}, y\right) = \sum_{k=1}^{\infty} \left[\phi_k\left(x^{\mu}\right) - \phi_{-k}\left(x^{\mu}\right)\right] sin\left(ky/R\right)$$

Bulk and Brane Fields

- Brane fields: they do not conserve the KK number in interactions involving bulk fields.
  They do not have KK towers and admit an expansion (in the fixed points of the orbifold)

$$\Phi\left(x^{\mu}, y\right) = \phi^{(A)}\delta(y) + \phi^{(B)}\delta(y - \pi R)$$



KK Modes in $D = 4 + n$ ***Dimensions***

Complex scalar field $\Phi(\mathbf{x}, \mathbf{y})$ in $D = 4 + n$ dimensions. It depends from the usual $D = 4$ coordinates $\mathbf{x} = (x^0, x^1, x^2, x^3)$ and from the extra space-like coordinates $\mathbf{y} = (y^1, \cdots, y^n)$ compactified over $S^1 \times \cdots \times S^1$ (same radius R).

Demanding periodicity under

$$y_i \to y_i + 2\pi R$$

we have

$$\Phi(\mathbf{x}, \mathbf{y}) = \sum_{k_1=-\infty}^{\infty} \cdots \sum_{k_n=-\infty}^{\infty} \Phi_{(\mathbf{k})}\,(\mathbf{x})\, exp\,(i\mathbf{k} \cdot \mathbf{y}/R)$$

where $\mathbf{k} = (k_1, \cdots, k_n)$ with $k_i \in \mathbb{Z}$.

KK Modes in $D = 4 + n$ Dimensions

In general the mass of each KK mode is given by

$$m_k^2 = m_0^2 + \frac{\mathbf{k} \cdot \mathbf{k}}{R^2}$$

where m_0 is the mass of the 0-mode.

In the case in which the radii are different from each other with denote in general

$$R^2 = \sum_{i=1}^{n} R_i^2$$

Warped Extra Dimensions

When you take into account energy density on the brane you have to introduce a second brane in order to obtain a stable configuration.

The $D = 5$ metric on the brane is no more flat.

Demanding $D = 4$ Lorentzian (flat) metric

$$ds^2 = g_{AB} dx^A dx^B = \omega(y) \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$$

where $\omega(y) = e^{-\beta(y)}$ and the $D = 5$ metric is conformally flat

Warped Extra Dimensions