Initial Correlation Effects in Time-Dependent Transport with One-Dimensional Interacting Leads

E. Perfetto¹,*, G. Stefanucci¹,², and M. Cini¹,³

¹Dipartimento di Fisica, Università di Roma Tor Vergata, Via della Ricerca Scientifica 1, I-00133 Rome, Italy
²European Theoretical Spectroscopy Facility (ETSF), Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Unità Tor Vergata, Via della Ricerca Scientifica 1, 00133 Rome, Italy

We study theoretically the memory effects due to different kinds of initial conditions in the transport properties of one-dimensional systems described by the Tomonaga-Luttinger model. We show that in presence of electron–electron interactions the sudden switching of a weak link between two initially uncontacted reservoirs induces a qualitative change in the transient current with respect to the contacted case. In particular the different switching process produces a change in the power-law temporal relaxation of the current towards the steady-state as well as a significant suppression of the transient oscillations. Even more dramatic is the response to a sudden interaction quench, which remarkably leads to a current–voltage characteristic differing from the one displayed if the system was initially interacting.

Keywords: Luttinger Liquids, Quantum Quenches, Time-Dependent Transport.

The theoretical investigation of time dependent phenomena in nanoscale systems is receiving growing attention due to the recent progress in the miniaturization of ultrafast electron devices.¹ For practical applications, it is crucial to achieve full control of the transient dynamics of these systems after the sudden switching of an external perturbation.²–¹¹ This has triggered the interest to the so called preparative nonequilibrium properties, i.e., the study of the temporal evolution of nanoscopic devices initially prepared in a given configuration. In these systems the transport problem is usually addressed within the partitioned approach, which consists in switching the contacts between the reservoirs at time \( t = 0 \).¹² Thus the intriguing question arises: how the properties developed in this way change if the reservoirs were contacted already at the equilibrium? If the system is non-interacting, it is well known that the switching process of the contacts only affects the early transient dynamics, and quantities like current and density relax towards the same steady-state value.¹²–¹⁵ In this paper we show that the electron–electron interactions change dramatically such a scenario, and also produce striking memory effects when different kinds of sudden perturbations are considered.

We consider two one-dimensional interacting leads described as Luttinger Liquids (LL),¹⁶ see Figure 1. Despite the large amount of work devoted to study steady-state properties, the time-dependent (TD) transport of LL has been so far poorly investigated.¹⁷,¹⁸ Our work is motivated by the fact that when particles interact in the whole system, the electron liquid does not necessarily relax to the ground state after an interaction quench (thermalization breakdown).¹⁹–²¹ Focussing on the intrinsically nonequilibrium transport problem, we ask whether the sudden switching of a given perturbation can generate a memory effect on the flowing current or not. We study two different kinds of perturbations, namely the sudden switching of a weak link between two interacting reservoirs and the sudden switching of the electron–electron interactions. In the first case we show that the switching process changes qualitatively the transient dynamics, producing a variation in the power-law temporal relaxation towards the same steady-state; in the second case the effect is even more dramatic, since the sudden interaction quench produces also a change in the current–voltage characteristic.

The equilibrium Hamiltonian for the system of Figure 1 reads

\[
H_0 = H_R + H_L + \eta L H_I + \eta R H_T. \tag{1}
\]

The one-body part of the left (\( L \)) and right (\( R \)) leads is

\[
H_{R/L} = \mp iv F \int_{-\infty}^{\infty} dx \phi_{R/L}^T(x) \partial_x \phi_{R/L}(x) \tag{2}
\]
where the fermion field $\psi_{R,L}$ describes right/left moving electrons in $R/L$ lead with Fermi velocity $v_F$ (chiral leads).

We take a density–density interaction of the form

$$H_1 = \frac{1}{2} \int_{-\infty}^{\infty} dx \left[ 2g_c \rho_R(x) \rho_L(x) + g_g(\rho^2_R(x) + \rho^2_L(x)) \right],$$

where $\rho_{R,L} = \langle \psi_{R,L}^\dagger \psi_{R,L} \rangle$ is the normally ordered fermionic density operator relative to the Fermi sea, and $g_c/g_g$ are the forward scattering couplings, corresponding to inter/intra lead interactions respectively. The two chiral LLs are linked at $x = 0$ via the tunneling term

$$H_T = \lambda \psi_R^\dagger(0) \psi_L(0) + \text{H.c.},$$

which does not commute with the total number of electrons $N_{R,L}$ of each lead.\(^{22}\)

If a bias $V = V_L - V_R$ is established between the leads at, say, time $t = 0$, a finite current $j(t)$ starts flowing across the link. The current operator (in atomic units) $J = dN_G/dt = -dN_R/dt$ reads

$$J = i\lambda \psi_R^\dagger(0) \psi_L(0) + \text{H.c.}$$

The current $j(t)$ is the TD average of $J$ over the ground state $|\Psi_0\rangle$ of $H_0$, i.e.,

$$j(t) = \langle \Psi_0 | J_R(t) | \Psi_0 \rangle$$

where $J_R(t)$ is the $J$ operator in the Heisenberg representation with respect to the interacting, contacted and biased Hamiltonian $H = H_0 + H_R + H_L + H_T$ in which $H_T = V_{\alpha} N_L + V_{\beta} N_R$. Notice that different factors $\eta_0 = 0, 1$ and $\eta_T = 0, 1$ yields different forms of $H_0$ and hence different initial states $|\Psi_0\rangle$. At positive times the Hamiltonian is the same in all cases.

We start our analysis by calculating $j(t)$ when $\eta_T = 0$ (initially uncontacted case) and $g_c = g_g = 0$ (always non-interacting system). In terms of the Fourier transform $\psi_{R,L}^\dagger$ of the original fermion fields, the current operator in Eq. (5) reads $J = (i\lambda/a) \sum_{\alpha \neq \beta} \psi_{\alpha}^\dagger \psi_{\beta}^\dagger + \text{H.c.}$, with $a$ the usual short-distance cutoff. Its expectation value is then

$$j(t) = 2\lambda \text{Re} \sum_{\alpha \in R,L} \int \frac{dp}{2\pi} \bar{f}_p^{\alpha}(t) f_p^{\alpha}(t) \Gamma^{\alpha\alpha}(t)$$

where $f_p^{\alpha} = f(\alpha v_F p)$ is the Fermi function of the lead $\alpha$ and

$$\Gamma^{\alpha\beta}(t) = -ia \int \frac{dk}{2\pi} \langle \Psi_0 | \psi_{\alpha}^\dagger e^{-iH_0 t} \psi_{\beta} | \Psi_0 \rangle$$

is the sum of the probability amplitudes (retarded Green’s functions) for the transition from an excitation with momentum $p$ in lead $\beta$ to an excitation with momentum $k$ in lead $\alpha$. From the Dyson equation it is straightforward to find

$$\Gamma^{\alpha\alpha}(t) = -ie^{i\pi/2} p V/(1 + c^2),$$

with $c = \lambda/(2v_F)$, and upon substitution in Eq. (7)

$$j(t) = \frac{2c^2}{\pi(1 + c^2)^2} V.$$

The current is discontinuous in time; the steady-state value is reached instantaneously. This is due to the unbound (relativistic) energy spectrum and the lack of interactions. As we shall see, when $H_T \neq 0$ the transient regime is more complex.

The problem does not have an exact solution when both $H_0$ and $H_T$ are present. Below, we calculate $j(t)$ to lowest order in $\lambda$. In general, perturbative treatments of the tunneling amplitude are a delicate issue.\(^{23,24}\) In our case, $j(t)$ has a Taylor expansion with convergence radius $\lambda < 2v_F$ for $H_T = 0$, see Eq. (10). We, therefore, expect a finite convergence radius at least for small interaction strengths. Let the unperturbed Hamiltonian be $H_0 = H_R + H_L + \eta_0 H_T$ in equilibrium ($t < 0$) and $H_t = H_R + H_L + H_T + H_V$ at positive times. To the lowest order in $\lambda$ we find

$$j(t) = i\langle \bar{\Psi}_0 \rangle \int_0^t ds \left[ H_T, \bar{J}_R(s) \right] \bar{J}_R(t) \left[ H_T, \bar{J}_R(-\tau) \right] |\bar{\Psi}_0\rangle$$

with $|\bar{\Psi}_0\rangle$ the ground state of $\bar{H}_0$. The first term in the r.h.s. is the standard Kubo formula. Such term alone describes the transient response when the contacts are switched on at $t = 0$ ($\eta_T = 0$). If, however, the equilibrium system is already contacted ($\eta_T = 1$) we must introduce a correction; this is the physical content of the second term.\(^{25}\) At any finite time initial correlation effects are visible in both terms due to the ground state dependence on $\eta_T$. When $t \rightarrow \infty$ only the Kubo term survives, which yields the steady-state current $j_s$. The dependence of $j_s$ on the ground state ($\eta_T = 0, 1$) is addressed below.

The averages in Eq. (11) can be explicitly calculated by resorting to the bosonization method.\(^{16}\) We define the scalar fields $\phi$ and $\theta$ from

$$\rho_{R}(x) + \rho_{L}(x) = \frac{1}{\sqrt{\pi}} \delta_{\phi} \phi(x),$$

$$\psi_{R,L}(x) = \frac{\kappa_{R,L}}{\sqrt{2\pi a}} e^{i\tau \phi}$$

with $\kappa_{R,L}$ the anticommuting Klein factors. The scalar fields obey the commutation relation $[\phi(x), \theta(x')] = i\text{sgn}(x - x')/2$. In terms of $\phi$ and $\theta$ the Hamiltonian $H = H_R + H_L + H_T$ has the simple quadratic form

$\text{Fig. 1. Sketch of the device. Two interacting leads hosting } L \text{ and } R \text{ movers are connected at } x = 0 \text{ via a weak link. A bias voltage } V_L - V_R \text{ can be applied between the leads.}$
\[ H = \frac{v}{2} \int_{-\infty}^{\infty} dx \left[ K^{-1} (\partial_x \phi(x))^2 + K \partial_x \theta(x) \right] \]  

(13)

with \( v = \sqrt{(2\pi v_F + g_\theta)^2 - g_\lambda^2}/2\pi \) the renormalized velocity and \( K = \sqrt{(2\pi v_F + g_\theta - g_\lambda)/(2\pi v_F + g_\theta + g_\lambda)} \) a parameter which measures the interaction strength. Notice that \( 0 < K \leq 1 \) for repulsive interactions; \( K \gg 1 \) corresponds to the noninteracting case while small values of \( K \) indicate a strongly correlated regime. The great advantage of the bosonization is that the averages over \( |\psi_0\rangle \) become averages over the bosonic vacuum, i.e., the vacuum of the normal modes of \( \phi \) and \( \theta \).

By employing the gauge transformation \( \psi_{LR} \rightarrow \psi_{LR} e^{iv_\psi x t} \) the problem of evaluating Eq. (11) is reduced to the calculation of different vacuum averages. After some tedious algebra one finds

\[ J(t) = \xi \text{Re} [\eta_1 A_1(t) + B_0(t)] \]  

(14)

where \( \xi = \lambda^2/(\pi a)^2 \) while

\[ A_0(t) = \int_0^\infty d\tau \sin(V\tau) \left[ \frac{a}{a - iv(t + i\tau)} \right]^2 \]  

\[ B_0(t) = i \int_0^t ds \sin[V(s - t)] \times \left[ \frac{a^2}{a^2 - v^2(s - t)^2} \right]^{(1 + \kappa)/2} \times \left[ \frac{(a^2 + 4v^2 s^2)/(a^2 + v^2 s^2)}{[a^2 + v^2(s + t)^2]^2} \right]^{(1 - \kappa)/4} \]  

(15)

for \( \eta = 0 \) and

\[ A_1(t) = \int_0^\infty d\tau \sin(V\tau) \left[ \frac{a}{a - iv(t + i\tau)} \right]^{2\kappa} \]  

\[ B_1(t) = i \int_0^t ds \sin[V(s - t)] \left[ \frac{a}{a - iv(t - s)} \right]^{2\kappa} \]  

(16)

for \( \eta = 1 \). Equations (14–16) and the subsequent discussion constitute the main result of the present work. In all cases \( (\eta_1, \eta_1 = 0, 1) \) \( J(t) \) is an odd function of \( V \), as it should be. We also notice that for noninteracting systems \( (K = 1) \) we recover the expected result \( A_1 = A_0 \) and \( B_1 = B_0 \). In this case the function \( \xi \text{Re}[B_1, a] \) coincides with the current in Eq. (10) to the lowest order in \( \lambda \). We can now provide a quantitative analysis of the TD current response for different preparative configurations.

We consider an initially contacted \( (\eta_1 = 1) \) and uncontacted \( (\eta_1 = 0) \) correlated ground state \( (\eta_1 = 1) \) and compare the corresponding TD currents \( j_{11} \equiv \xi \text{Re}[A_1, B_1] \) and \( j_{10} \equiv \xi \text{Re}[B_1] \). The current \( j_{10}(t) \) has been recently computed in Ref. [18] and it is proportional to the generalized hypergeometric function. In the long time limit it returns the well known steady-state result \( j_{10}(t \rightarrow \infty) \propto v^{2K - 1} \) that was obtained long ago by Kane and Fisher.\(^27\) Since \( A_1(t \rightarrow \infty) \propto 0 \), \( j_{11} \) approaches the same steady state. Notice that the small bias limit is ill-defined for \( 0 < K < 1/2 \) due to the break down of the perturbative expansion in powers of \( \lambda \).\(^26,28\) Interestingly, such a pathology does not appear for an uncorrelated ground state \( (\eta_1 = 0) \), see below. Even though \( j_{10}(t \rightarrow \infty) \) \( j_{11}(t \rightarrow \infty) \) the relaxation is different in the two cases, see Figure 2. The function \( j_{10}(t) \) approaches the asymptotic limit with transient oscillations of frequency \( V \) and damping envelope proportional to \( t^{-2K} \). The more physical current \( j_{11} \), instead, decays much slower. The integral in \( A_2(t) \) can be calculated analytically to give

\[ j_{11}(t) - j_{10}(t) = \xi a^{2\kappa} \sin(Vt) \cos((1 - 2K) \arctan(vt/a)) \]  

\[ 2v(2K - 1)(a^2 + v^2 t^2)^{K - 1/2} \]  

(17)

which for long times decays as \( t^{1-2K} \). (Equation (17) provides an independent, TD evidence that the perturbative treatment breaks down for \( 0 < K < 1/2 \)). Thus, an initially contacted state changes the power-law decay from \( t^{-2K} \) to the slower behavior \( \sim t^{-2K} \). The amplitude of the transient oscillations is also significantly different, due to the factor \( (1 - 2K)^{-1} \) in Eq. (17). For \( K = 0.6 \), \( j_{11} \), oscillates with an amplitude more than 10 times larger than that of \( j_{10} \), see Figure 2. The magnification of the oscillations was unexpected since for \( j_{11} \), we only switch a bias while for \( j_{10} \) we also switch the contacts. We verified (not reported) that this is a genuine effect and not an artifact of the perturbative treatment. In fact, for \( \eta_1 = \eta_1 \) and zero bias the density matrix \( \rho(t) = (\Psi_0 | \Psi_{LR}(t) \rangle \langle \Psi_{LR}(t) | \Psi_0) \) does not evolve in time to first order in \( \lambda \) (this is obviously true for the exact density matrix). The constant value \( \rho(t) = \rho(0) \) is the result of a subtle cancellation of TD functions similar to \( A_1(t) \) and \( B_1(t) \).

Next we consider the effects of correlations in the ground state. We take \( \eta_1 = 1 \) and compare the TD currents \( j_{11} \) and \( j_{10} \) resulting from Eq. (14) when \( \eta_1 = 1 \) and \( \eta_1 = 0 \) respectively. Notice that \( j_{11} \approx j_{11} \) (as already calculated above). The current \( j_{10} = \xi \text{Re}[A_1 + B_1] \) is the response to a sudden bias switching and interaction quench; at \( t > 0 \) the transient currents \( j_{10}(t) \) (solid curve) and \( j_{11}(t) \) (dashed curve) for \( V = 10^{-2}, K = 0.6 \). In the long-time limit they reach the same steady-state value. Current is in units of \( \xi a/v \), \( V \) is in units of \( v/a \) and \( t \) is in units of \( a/v \).

\[ \text{Fig. 2.} \]
electrons start tunneling from $L$ to $R$ and at the same time forming interacting quasiparticles. The interaction quench has a dramatic impact on the transport properties, both in the transient and steady-state regimes. From Figure 3 we clearly see that the relaxation behavior is different. The damping envelope of $j_0(t)$ is proportional to $t^{-K^2 - 1}$ as opposed to $t^{-1}$ of $j_1(t)$. Notice that the exponent $-K^2 - 1$ is strictly negative for all $K$ (the perturbative expansion in powers of $\lambda$ is meaningful for all $K$).

In the long-time limit we obtain the current–voltage characteristic

$$\lim_{t \to \infty} j_0(t) = \kappa \text{sgn}(V)|V|^{K^2}$$

with $\kappa = -\xi(a/\nu)^{K^2+1} \Gamma(-K^2) \sin(\pi K) \sin(K^2 \pi/2)$. The nonohmic behavior $\sim V^{K^2}$ differs from $\sim V^{2K-1}$ exhibited by $j_1$. The dependence of the steady-state values of $j_0$ and $j_1$ on bias $V$ and interaction strength $K$ is illustrated in Figure 4. We thus found that ground state correlations are not reproducible by quenching the interaction. The system remembers them forever and steady-state quantities are inevitably affected. This behavior is reminiscent of the thermalization breakdown enlightened by Cazalilla and others. Here, however, we are neither in equilibrium nor close to it; the TD bias is treated to all orders. The non-equilibrium exponents of the current–voltage characteristics refer to current-carrying excited states as obtained from the full TD Schrödinger equation with different initial states. It is noteworthy that for $\eta = 0$ the small bias limit is well-behaved for all $K$ since $j_0(t \to \infty) \propto V^{K^2}$ (independent evidence of the validity of the perturbative treatment).

In conclusion we studied the role of different preparative configurations in TD quantum transport between LLSs. First we showed that the currents are analytic functions of $\lambda$ at least for small interaction strengths. By using bosonization methods we proved that initial contacts do not change the steady-state but significantly alter the transient behavior, changing the damping envelope from $\sim t^{-2K}$ to $\sim t^{-1}$ and magnifying the amplitude of the oscillations. The effects of initial correlations are even more striking. Besides a different power law decay ($\sim t^{-2K}$ versus $\sim t^{-K^2 - 1}$ damping envelope) the steady-state current is also different; the current–voltage characteristic changes from $j \propto V^{2K-1}$ to $j \propto V^{K^2}$. These findings suggest the occurrence of a complex entanglement between equilibrium and non-equilibrium properties of strongly confined interacting electrons. Our predictions could be experimentally confirmed by transport measurements of ultracold fermionic atoms loaded in optical lattices, recently proposed as candidates to realize highly controllable and tunable LLSs.

References and Notes

15. This is rigorously true provided that no bound states are present, see G. Stefanucci, Phys. Rev. B 75, 195115 (2007).

Fig. 4. Contour plot of the steady-state value of $j_1$ (left panel) and $j_0$ (right panel) as a function of $V$ and $K$. Units are the same as in Figure 2.

Fig. 3. Transient currents $j_0(t)$ (solid curve) and $j_1(t)$ (dashed curve) for $V = 10^{-2}$, $K = 0.7$. In the long-time limit they reach different steady-state values. Same units as in Figure 2 are used.
Perfetto et al. Initial Correlation Effects in Time-Dependent Transport with One-Dimensional Interacting Leads

22. The Hamiltonian $H_R + H_L + H_I + H_T$ can be used to describe, e.g., tunneling effects between two disconnected quantum Hall liquids when a gate voltage is applied in the vicinity of their boundaries, see C. de C. Chamon, D. E. Freed, and X. G. Wen, *Phys. Rev. B* 51, 2363 (1995) and A. M. Chang, *Rev. Mod. Phys.* 75, 1449 (2003).
25. Such term stems from first-order perturbation theory along the imaginary Matzubara time axis. In the Green’s function language it is related to mixed Greens functions with one real and one imaginary time argument, see Refs. [13, 3].

Received: 7 December 2010. Accepted: 3 January 2011.