Time-resolved charge fractionalization in inhomogeneous Luttinger liquids

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The recent observation of charge fractionalization in single Tomonaga-Luttinger liquids (TLLs) [H. Kamata et al., Nat. Nanotechnol. 9, 177 (2014)] opens new routes for a systematic investigation of this exotic quantum phenomenon. In this Rapid Communication we perform measurements on two adjacent TLLs and put forward an accurate theoretical framework to address the experiments. The theory is based on the plasmon scattering approach and can deal with injected charge pulses of arbitrary shape in TLL regions. We accurately reproduce and interpret the time-resolved multiple fractionalization events in both single and double TLLs. The effect of intercorrelations between the two TLLs is also discussed.

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Introduction. When electrons are confined in one spatial dimension the traditional concept of Fermi-liquid quasiparticles breaks down [1–3]. The Fermi surface collapses and the elementary excitations become collective modes of bosonic nature [4]; these are two distinctive features of the so-called Tomonaga-Luttinger liquid (TLL) [5,6]. A paradigmatic example of TLL is the edge state of a quantum Hall system, typically created on contiguous boundaries of two-dimensional semiconductor heterostructures [7]. Here the properties of the TLL can be tuned by varying the gate voltage [8], the magnetic field, the filling factor \( \nu \) [7], and electrostatic environment of the channel [9,10]. Spatially separated TLLs with opposite chirality can be realized in systems with \( \nu > 1 \), and as a result of strong correlations, charge fractionalization occurs [11,12]. According to the plasmon scattering theory [13,14] an electron injected into a TLL region undergoes multiple reflections from one edge of the sample to the other. A fraction \( r \) (dependent on the TLL parameter \( g \)) of the injected charge \( Q \) is reflected back in the adjacent edge, and the remaining fraction \( 1 − r \) is transmitted forward through the same edge. This fractionalization is a transient effect [13–21]. Due to charge compensations occurring at every fractionalization a full charge \( Q \) is transmitted in the long-time limit. Therefore, only time-resolved (or finite frequency) experiments could detect the value of the fractional charge \( rQ \). The first conclusive evidence of transient fractionalization was reported only recently by means of time-resolved transport measurements of charge wave packets [22]. This provides complementary evidence of fractionalization seen in shot-noise measurements [19–21,23], frequency-domain experiments [24], and momentum-resolved spectroscopy [25].

In this Rapid Communication we implement the technique developed in Ref. [22] to perform transport measurements across two spatially separated TLLs and highlight the effect of inter-TLL interactions. Furthermore, we put forward a theoretical framework to calculate the evolution of wave packets of arbitrary shape scattering against multiple noninteracting-liquid/TLL interfaces arranged in different geometries. By a proper treatment of the boundary conditions we are able to make direct comparisons with the measured signal. All features of the transient current are correctly captured both in the single and double TLL systems.

Experimental setup. Figure 1 shows the sample patterned on a GaAs/AlGaAs heterostructure with chiral one-dimensional edge channels formed along the edge of the two-dimensional electronic system (2DES) in a strong perpendicular magnetic field \( B \). Artificial TLL can be formed in a pair of counter-propagating edge channels along both sides of a narrow gate metal [22]. Other unpaired channels are considered as noninteracting (NI) leads. Two types of TLL regions were investigated: type-I TLL, with NI leads on both ends, and type-II TLL, with NI leads only on the left and a closed end on the right. We can selectively activate one or both the TLL regions by applying appropriate voltages \( (V_{G1} \text{ and } V_{G2}) \). A nonequilibrium charge wave packet of charge \( Q \approx 150e \) is generated by depleting electrons around an injection gate with a voltage step applied on the gate. The wave packet travels along a NI lead as shown in Fig. 1, and undergoes charge fractionalization processes at the left and right ends of the TLL regions. The multiple charge fractionalization processes must be investigated separately. The reflected wave packet appears on another NI lead, on which a time-resolved charge detection scheme is applied with a quantum point contact (QPC) detector [8]. We have successfully resolved the reflected wave packets of charge \( Q^{(\text{refl})}_1 \) fractionalized at the left boundary and \( Q^{(\text{refl})}_2 \) at the right boundary. Typical wave forms are shown by dots in Figs. 3 and 4. The fractionalization ratio \( r \), which is related to the TLL parameter \( g \) through \( g = (1 − r)/(1 + r) \), can be extracted from \( r = Q^{(\text{refl})}_2/Q \) and is found to be approximately \( g = 0.92 \) [22]. The charge velocity in the TLL region can be measured from the time interval between the two reflected wave packets. The interest in activating both type-I and -II regions is to assess the role of the long-range Coulomb interaction between the two TLLs.

Model and formalism. To model the setup of Fig. 1 we consider two parallel chiral edges hosting right- (\( R \)) and left- (\( L \)) moving electrons (see Fig. 2). Electrons with opposite chirality experience a space-dependent repulsion \( V(x) \). In the
regions where \( V(x) = 0 \) we have a NI liquid and otherwise, \( V(x) = V \), a TLL is formed. For electrons with the same chirality an additional repulsion \( U(x) = U \) in the NI liquid and \( U(x) = U^* \) in the TLL is included. Spatial inhomogeneities in \( V(x) \) induce backscattering from the \( R \) to the \( L \) edge (and vice versa) even without an interedge hopping [13,14]. The low-energy Hamiltonian of the system reads [7]

\[
\hat{H} = \sum_{\alpha=L,R} i\alpha v_F \int dx \left( \hat{\psi}_\alpha^\dagger(x) \hat{\partial}_x \hat{\psi}_\alpha(x) + 2\pi \int dx \left\{ V(x) \hat{n}_R(x) \hat{n}_L(x) + \frac{U(x)}{2} \left[ \hat{\psi}_L^2(x) + \hat{\psi}_R^2(x) \right] \right\} \right),
\]

where the fermion field \( \hat{\psi}_R^{(\dagger)}(x) \) destroys (creates) \( R / L \) edge-state electrons moving with bare Fermi velocity \( \alpha v_F \equiv \pm v_F \), and \( \hat{n}_\alpha \equiv \hat{\psi}_\alpha^\dagger \hat{\psi}_\alpha \) is the density fluctuation operator. For a nonperturbative treatment of the interaction we bosonize the field operators as \( \psi_\alpha(x) = \frac{\eta_\alpha}{\sqrt{v_F}} e^{-\pi i/\sqrt{2}} \phi_\alpha(x) \), with \( \eta_\alpha \) the anticommuting Klein factor, \( a \) a short-distance cutoff, and \( \phi_\alpha(x) \) the chiral boson fields. The density can then be expressed as \( \hat{n}_\alpha = -\partial_x \phi_\alpha / \sqrt{\pi} \). Introducing the auxiliary fields \( \hat{\phi} = \phi_L + \hat{\phi}_R \) and \( \hat{\theta} = \phi_L - \phi_R \), Eq. (1) becomes [1]

\[
\hat{H} = \frac{1}{2} \int dx \left\{ \frac{\hat{v}(x)}{g(x)} (\hat{\partial}_x \phi(x))^2 + \hat{v}(x) g(x) (\hat{\partial}_x \phi(x))^2 \right\},
\]

where for a TLL region of length \( \ell \) the parameter \( g(x) \) and the renormalized velocity \( \hat{v}(x) \) depend on the interactions through the relations

\[
g(x) = \begin{cases} \frac{\sqrt{v_F + U^*/2} - \sqrt{V}}{\sqrt{v_F + U^*/2} + \sqrt{V}} & \text{for } 0 < x < \ell \\ 1 & \text{otherwise}, \end{cases}
\]

\[
\hat{v}(x) = \begin{cases} \sqrt{(v_F + U^*)^2 - V} \equiv \hat{v}^* & \text{for } 0 < x < \ell \\ v_F + U \equiv \hat{v} & \text{otherwise}. \end{cases}
\]

The temporal evolution of the system is governed by the equation of motion for \( \phi \) [26]. Taking the average \( \langle \phi(x,t) \rangle \equiv \langle \phi(x,t) \rangle \) over an arbitrary wave-packet state we find

\[
\frac{d^2}{dt^2} \phi(x,t) = v(x) g(x) \partial_x \left( \frac{\hat{v}(x)}{g(x)} \hat{\partial}_x \phi(x,t) \right),
\]

which implies that \( \phi \) and \( \frac{\hat{v}(x)}{g(x)} \hat{\partial}_x \phi \) are continuous for all \( x \). For independent channels, as those of the type-I geometry illustrated in Fig. 2, these are the only conditions to impose on the solution of Eq. (4) [10,13,27]. On the other hand, for the type-II geometry one has to further impose that \( R \) electrons are converted into \( L \) electrons and vice versa, i.e., that the channels are not independent. The proper treatment of boundary conditions, absent in previous works, leads to a qualitatively different transient fractionalization since the transmission and reflection coefficients are entangled. Once \( \phi(x,t) \) is known the total density and current are extracted from \( \rho(x,t) = e^i(\hat{h}(x,t)) = -e\hat{\partial}_x \phi(x,t) / \sqrt{\pi} \) and \( j(x,t) = e^i(\hat{\partial}_x \phi(x,t)) / 4\pi \).

We consider an incident wave packet injected in the upper \( R \) edge (see Fig. 2). Then the solution of Eq. (4) can be expanded in right-moving scattering states \( s_q(x) \) of energy \( \epsilon_q = vq \) according to \( \phi(x,t) = \int_{-\infty}^{\infty} \frac{dq}{2\pi} \phi_q s_q(x) e^{-\epsilon_q t} \) [28]. For
a wave packet initially, say at time $t = 0$, localized in $x < 0$ the function $\phi_q$ is related to the Fourier transform $\rho_q^{(\text{inc})}$ of $\rho^{(\text{inc})}(x) = \rho(x, 0)$ by the relation $\phi_q = \frac{i\sqrt{\pi}}{q} \rho_q^{(\text{inc})}$ [29]. Therefore, once $\rho_q(x)$ is known the time-dependent density and current are given by

$$
\rho(x, t) = -i \int_{-\infty}^{\infty} \frac{dq}{2\pi} \rho_q^{(\text{inc})} e^{-iqt} \phi_q(x),
$$

$$
\jmath(x, t) = v \int_{-\infty}^{\infty} \frac{dq}{2\pi} \rho_q^{(\text{inc})} e^{-iqt} \phi_q(x),
$$

Below we solve the scattering problem in the geometries of the experiment.

Type-I geometry. This geometry is illustrated in Fig. 2(a) and has been realized in Ref. [22]. We look for scattering states of the form

$$
s_q(x) = \begin{cases} 
    e^{iqx} + r_q e^{-iqx} & \text{for } x < 0 \\
    a_q e^{iqx} + b_q e^{-iqx} & \text{for } x > \ell ,
\end{cases}
$$

with $q' = \frac{q}{g}$. By imposing the continuity conditions at the boundaries we obtain a $4 \times 4$ linear system [26] that we solve exactly. If we are interested in the current detected at the left boundary and a fractionalized charge $\rho$ at the right boundary the total reflected charge vanishes $-r \rho + b \rho^{(\text{inc})} - a \rho^{(\text{inc})} + b^{(\text{inc})}$.

The comparison with the experiment we acquire the transient nature of the fractionalization phenomenon. For the best fitting as shown in Fig. 3 the agreement with the current calculated from Eq. (8) is remarkably good.

Type-II geometry. Here a single edge is bent on itself as illustrated in Fig. 2(b). Therefore $R$ electrons in the upper branch are converted in $L$ electrons in the lower branch. We model this geometry by imposing that the $L$ amplitude $b_q$ of the scattering state in the TLL region equals $-d_q e^{2iq\ell} [26]$. Following the same line of reasoning as before we find the reflection coefficient

$$
r_q = -r + 4g \sum_{n=1}^{\infty} \xi_n e^{2in\ell} q',
$$

where $g = 1 \pm g$, $r = \frac{\xi}{g^*}$, and $\xi = e^{\pi - i\pi} [26]$. Inserting this expression in Eq. (5) the time-dependent density and current for $x < 0$ read [31]

$$
\rho(x, t) = \rho^{(\text{inc})}(x-) + \rho^{(\text{refl})}(x+),
$$

$$
\jmath(x, t) = v[\rho^{(\text{inc})}(x-) - \rho^{(\text{refl})}(x+)],
$$

with $x_{\pm} = x \pm vt$, $x_n = \frac{2\ell n}{1 - v^2}$, and

$$
\rho^{(\text{refl})}(x+) = r \rho^{(\text{inc})}(-x+) - 4g \sum_{n=1}^{\infty} \xi_n \rho^{(\text{inc})}(-x+ + x_n).
$$

Equation (9) generalizes the result of Ref. [13] to arbitrary wave-packet shapes. The first reflection occurs at time $t_1 = \frac{2\ell}{v}$ (from the initial position of the wave packet) at the left boundary and a fractionalized charge $Q_2^{(\text{refl})}$ is reflected back in the $L$ edge [here $Q = \int dx \rho^{(\text{inc})}(x)$]. The transmitted fractional charge propagates in the TLL region, a second reflection occurs at the right boundary, and at time $t_2 = t_1 + 2\ell/v^2$ a second wave packet of charge $Q_2^{(\text{refl})} = -Q(4gg^* g^*_L) = -Q(1 - r^2)$ appears in the $L$ edge. The fractionalization sequence continues ad infinitum and the reflected charge $Q_n^{(\text{refl})}$ diminishes at each event. At the end of the infinite sequence the total reflected charge vanishes since $Q^{(\text{refl})} = \sum_{n=1}^{\infty} Q_n^{(\text{refl})} = -r - 4g \sum_{n=1}^{\infty} \xi_n = 0$. This is a consequence of the chiral charge conservation and highlights the transient nature of the fractionalization phenomenon. For the comparison with the experiment we acquire $\rho^{(\text{inc})}(x_0 - vt)$ from Ref. [22] (see inset in Fig. 3) and use $g = 0.92$, $\ell = \ell_1 = 68 \mu$m, and $v^* = 150$ km/s and estimated $v$ by a
The calculated reflected current is shown in Fig. 5 for comparison. From \(\Delta t_{\text{II}} = \ell/t_2/V_{\text{II}}\) with \(\Delta t_{\text{II}} \approx 1.0\) ns and \(\Delta t_{\text{II}} \approx 0.5\) ns we estimated \(V_{\text{II}} \approx 136 \text{ km/s}, V_{\text{II}} \approx 320 \text{ km/s}\), and \(v\) by a best fitting. The value \(g = 0.92\) (black dashed curve) is probably too large as the additional peak and dip are almost invisible. We therefore repeated the calculation with \(g = 0.87\) (black solid curve) to match the height of the positive main peak and find that the additional peak and dip are correctly more pronounced. The physical justification of a smaller \(g\) is elaborated in the conclusions.

Conclusions. We extended the plasmon scattering approach to address the charge fractionalization phenomenon recently observed in artificial TLLs of different geometries [22]. The method allows us to monitor the temporal evolution of a charge wave packet in each chiral edge of the experimental setup, thus providing a tool for a direct comparison with the time-resolved transport measurement. Quantitative agreement between theory and experiment is obtained for the type-I and type-II geometries. We then performed new measurements in a double-TLL geometry and found indications that electron correlations are enhanced due to the repulsion between electrons in different TLLs. Our calculations neglect the inter-TLL repulsion and the enhancement of correlations is effectively accounted for by a reduced TLL parameter \(g\). The proper inclusion of the long-range interaction across the bulk two-dimensional electron gas is eventually required for the ultimate understanding of the transport properties of interacting edge channels. 

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[28] For an incident wave packet injected from \( x = -\infty \) in the lower \( L \) edge \( e^{-i\epsilon q t} \rightarrow e^{i\epsilon q t} \).
[29] The property that the expansion coefficients of \( \rho^{(inc)}(x) \) are the same in the scattering-state basis and in the plane-wave basis is crucial to perform the \( q \) integral in Eqs. (5). This property can be checked by calculating \( \rho^{(inc)}(x) \) for all \( x \) (see Ref. [26]).
[30] To evaluate \( \rho \) and \( j \) in \( x > 0 \) the expressions of \( a_q, b_q \), and \( t_q \) are needed [26].
[31] Within our convention the current \( j \) carried by an excess of left-moving electrons is negative. Thus in order to compare the theoretical results with the experiment in Ref. [22], in producing the plots we have to revert the “−” sign appearing in the second lines of Eqs. (8).