Correlation-Induced Memory Effects in Transport Properties of Low-Dimensional Systems

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We demonstrate the remnant presence of initial correlations in the steady-state electrical current $j_S$ flowing between low-dimensional interacting leads. The leads are described as Luttinger liquids and electrons can tunnel via a quantum point contact. We derive an analytic result for the time-dependent flow of $j_S$ as an explicit functional of the switching process and to establish that $j_S$ is history dependent for nonsmooth switchings.

The equilibrium Hamiltonian (see Fig. 1) reads
\[ H_0 = H_R + H_L + \eta_I H_I + \eta_T H_T. \] (1)
The one-body part of the left ($L$) and right ($R$) leads is
\[ H_{L(R)}(L) = \frac{-i}{\hbar} \int dx \psi^\dagger_R(x) \delta_x \psi_R(x), \]
where $\psi_R(x)$ describes right ($R$) leads is an electron field in lead $R$ (chiral leads). We take a density-density interaction of the form
\[ H_I = \frac{i}{\hbar} \int dx g_L \rho(x) \rho_L(x) + \frac{1}{\hbar} \int dx \rho_R(x) \rho_L(x), \]
where $\rho_R(x) = \psi_R^\dagger(x) \psi_L(x)$ and $g_L$ is the forward scattering coupling.

If a bias $V = V_L - V_R$ is applied at, say, time $t = 0$, a finite current $j(t)$ starts flowing across the link. The current operator (in atomic units) reads
\[ J = i \lambda \psi^\dagger_L(0) \psi_L(0) + H.c. \]

At zero temperature $T$ the TD average of $J$ over the GS $|\Psi_0\rangle$ of $H_0$, i.e.,
\[ j(t) = \langle \Psi_0 | J(t) | \Psi_0 \rangle. \] (2)

where $J(t)$ is the $H_I(t)$ is the operator in the Heisenberg representation with respect to the interacting, contacted, and biased sequence of interaction quenches. We are able to write $j_S$ as an explicit functional of the switching process and to establish that $j_S$ is history dependent for nonsmooth switchings.

Nonequilibrium phenomena in open nanoscale systems offer a formidable challenge to modern science [1]. Controlling the electron dynamics of a molecular device is the ultimate goal of nanoelectronics and quantum computation [2]; its microscopic description a problem at the forefront of statistical quantum physics [3]. Resorting to approximate methods is inevitable to progress.

Standard many-body techniques consider an initial state with no interaction and with no contact between the system and the baths (leads from now on), and then switch them on in time [4–6]. In fact, it is plausible to believe that starting from the true interacting and contacted state the long-time results would not change. To what extent, however, is such belief actually the truth? This question is of both practical and fundamental interest. It has been shown [7] that for noninteracting electrons the initial contact plays no role at the steady state. Allowing for interactions in the system noninteracting electrons the initial contact plays no role at the steady state. Allowing for interactions in the system [8] found that steady-state quantities are not sensitive to initial correlations either. It is the purpose of this Letter to show that interacting leads change dramatically the picture: the switching process can indeed have a large impact on the relaxation and the steady-state behavior.

We consider two one-dimensional interacting leads described as Luttinger liquids (LL) [9], see Fig. 1. It is known that a LL does not relax to the ground state (GS) after a sudden quench of the interaction [10–12] (thermalization breakdown). The implications of such an important result in the context of time-dependent (TD) transport are totally unknown and will be here explored for the first time. We compare the dynamics of initially (a) contacted ($\eta_T = 1$) versus uncontacted ($\eta_T = 0$) and (b) interacting ($\eta_I = 1$) versus noninteracting ($\eta_I = 0$) LL when driven out of equilibrium by an external bias. Our main findings are that in (a) the system relaxes towards the same steady state although with a different power-law decay. In (b) the sudden quench of the interaction when $\eta_I = 0$ alters the steady current $j_S$ as well. This remains true for an arbitrary
Hamiltonian $H_1 = H_L + H_R + H_I + H_T + H_V$, $H_V = \sum_{a=L,R} \int dx \psi_a^*(x) \psi_a(x)$. Note that the factors $\eta_I$, $\eta_T$ in Eq. (1) refer to times $t < 0$ and different values $\eta_I$, $\eta_T$, $\eta_I = 0$, 1 yields different $H_0$ and hence different initial states $|\Psi_0\rangle$. At positive the Hamiltonian is the same in all cases.

The exact noninteracting solution.—We start our analysis by calculating $j(t)$ when $\eta_T = 0$ (initially uncontacted) and $g_2 = g_4 = 0$ (always noninteracting). In terms of the Fourier transform $\psi_{k_R}$ of the original fermion fields, the current operator reads $J = (i\lambda/\sigma) \sum_{k_R} \psi_{k_R}^\dagger \psi_{k_R} + H.c.$, with $\sigma$ the usual short-distance cutoff. Its expectation value is then

$$
j(t) = \lambda \Im \sum_{a=L,R} \int \frac{d^2k}{(2\pi)^2} \Gamma_p^a(t) f_{R}^{(p)}(\Gamma_p^a(t))^*$

where $f_{R}^{(p)}(\Gamma_p^a(t)) = f(\pm v_F p)$ is the Fermi function of lead $R(L)$ and $\Gamma_p^a(t) = -i a_f \int \frac{d^4k}{(2\pi)^4} \langle |\Psi_0\rangle \psi_{k_R}^\dagger e^{-iH_0 t} |\psi_{k_R}^\dagger |\Psi_0\rangle$ is the sum of the probability amplitudes for the transition $p_R \to k_R$. From the Dyson equation it is straightforward to find $\Gamma_p^a(t) = -ie^{i(\alpha v_F + \nu) t}/(1 + c^2)$ and $\Gamma_p^{a\dagger}(t) = -ic \Gamma_p^{a\dagger}(t)$, with $c = \lambda/(2v_F)$, and hence

$$
j(t) = \frac{2e^2}{\pi(1 + c^2)^2} V.

(3)

The current is discontinuous in time; the steady-state value is reached instantaneously. This is due to the unbound (relativistic) energy spectrum [5] and the lack of interactions, as discussed in detail in Ref. [13]. As we shall see, when $H_1 \neq 0$ the transient regime is more complex.

Current to lowest order in $\lambda$.—The problem does not have an exact solution when both $H_1$ and $H_T$ are present. Below, we calculate $j(t)$ to lowest order in $\lambda$. In general, perturbative treatments in the tunneling amplitude are a delicate issue. In our case $j(t)$ has a Taylor expansion with convergence radius $\lambda < 2v_F$ for $H_1 = 0$, see Eq. (3). We therefore, expect a finite convergence radius at least for small interaction strengths. Let the unperturbed Hamiltonian be $H_0 = H_R + H_L + \eta_I H_I$ in equilibrium ($t < 0$) and $H_1 = H_R + H_L + H_I + H_0$ at positive times. At zero temperature and to lowest order in $\lambda$

$$
j(t) = i\langle\tilde{\Psi}_0| \int_0^\infty ds [H_{T,R_1}(s), J_{R_1}(t)] - \eta_T \int_0^{-\infty} d\tau \times [H_{T,R_1}(\tau) J_{R_1}(t) + J_{R_1}(t) H_{T,R_1}(\tau) - H_{T,R_1}(t) H_{T,R_1}(\tau)] |\tilde{\Psi}_0\rangle,

(4)

with $|\tilde{\Psi}_0\rangle$ the GS of $H_0$. The first term in the right-hand side is the standard Kubo formula. Such term alone describes the transient response when the contacts are switched on at $t = 0$ ($\eta_T = 0$). If, however, the equilibrium system is already contacted ($\eta_T = 1$) we must account for a correction; this is the physical content of the second term [14]. At any finite time initial correlation effects are visible in both terms due to the dependence of $|\tilde{\Psi}_0\rangle$ on $\eta_I$. When $t \to \infty$ only the Kubo term survives, which yields the steady-current $j_S$. The dependence of $j_S$ on the initial state ($\eta_I = 0, 1$) will be addressed below.

The averages in Eq. (4) can be explicitly calculated by resorting to the bosonization method [9]. We introduce the scalar fields $\phi$ and $\theta$ from $\rho_R(x) + \rho_L(x) = \frac{1}{\sqrt{2\pi}} \delta(x, \phi(x)$ and $\psi_{R(L)}(x) = \frac{\kappa_{R(L)}}{\sqrt{2\pi a}} e^{i\sqrt{\kappa}(\phi(x) - \theta(x))}$, with $\kappa_{R(L)}$ the anticommuting Klein factors. In terms of $\phi$ and $\theta$ the Hamiltonian $H = H_R + H_L + H_I$ is a simple quadratic form $H = \frac{v}{2} \int dx K^{-1}(\partial_x \phi(x))^2 + K(\partial_x \theta(x))^2$, with $v = \sqrt{[2(2\pi v_F + g_4)^2 - g_2^2]/2\pi}$ the renormalized velocity and $K = \sqrt{[2(2\pi v_F + g_4 - g_2)/(2\pi v_F + g_4 + g_2)]}$ a parameter which measures the interaction strength. Note that $0 < K \leq 1$ for repulsive interactions and $K = 1$ corresponds to the noninteracting case.

By employing the gauge transformation [15] $\psi_{RL} \to e^{i\nu L/R} \psi_{RL}$ the problem of evaluating Eq. (4) is reduced to the calculation of bosonic vacuum averages [9]. After some tedious algebra one finds

$$
j(t) = \xi \Re \left[ \eta_T A_{\eta_I}(t) + B_{\eta_I}(t) \right],

(5)

where

$$
A_0(t) = \sin(Vt) \int_0^\infty d\tau \gamma^2(t + i\tau),

B_0(t) = i \int_0^\infty ds \left[ \left( \frac{\gamma^2(s + t)}{\gamma(s - t)} \right) - \left( \frac{\gamma^2(s - t)}{\gamma(s + t)} \right) \right]^{1 - K^2},

(6)

for $\eta_I = 0$ and

$$
A_1(t) = \sin(Vt) \int_0^\infty d\tau \gamma^2K(t + i\tau),

B_1(t) = i \int_0^\infty ds \sin(V(s - t)) \gamma^2K(s - t),

(7)

for $\eta_I = 1$, and where $\gamma(z) = a/(a - izv)$ and $\xi = \lambda^2/(\pi a)^2$. In all cases ($\eta_I = 0, 1$) $j(t)$ is an odd function of $V$, as it should be. We also notice that for noninteracting systems ($K = 1$) we recover the expected result $A_1 = A_0$ and $B_1 = B_0$. In this case the function $\xi \Re[B_{1,0}]$ coincides with the current in Eq. (3) to lowest order in $\lambda$. We can now provide a quantitative analysis of the TD current response for different preparative configurations.

Contacted versus uncontacted ground state.—We consider an initially contacted ($\eta_I = 1$) and uncontacted ($\eta_I = 0$) correlated GS ($\eta_I = 1$) and compare the corresponding TD currents $j_{T1} \equiv \xi \Re[A_1 + B_1]$ and $j_{T0} \equiv \xi \Re[B_1]$. The current $j_{T0}(t)$ has been recently computed in Ref. [16]. In the long-time limit it returns the well-known steady-state result

$$
j_S(\beta) = \sin(\pi K) \kappa(\beta) \text{sgn}(V) V |\beta|^{\beta}

(8)

with $\kappa(\beta) = -\xi (a/v)^{\beta + 1} \Gamma(-\beta) \sin(\beta\pi/2)$ and the exponent $\beta = 2K - 1$, obtained long ago by Kane and Fisher [17]. Since $A_1(t \to \infty) = 0$, $j_{T1}$ approaches the same steady state. Note that the small bias limit is ill defined.
for $K<1/2$ due to the breakdown of the perturbative expansion in powers of $\lambda$ [15,18]. Even though $j_{T_0}(t \to \infty) = j_{T_1}(t \to \infty)$ the relaxation is different in the two cases, see Fig. 2. The function $j_{T_0}(t)$ approaches the asymptotic limit with transient oscillations of frequency $V$ and damping envelope $\propto t^{-2K}$ [16]. The more physical current $j_{T_1}$, instead, decays much slower. The integral in $A_1(t)$ can be calculated analytically and yields

$$j_{T_1}(t) - j_{T_0}(t) = \xi a^2 V t \cos(2K - 1) \arctan(vt/a) \int_0^t \frac{d^2}{d\tau^2} \left[ \Delta(\tau) \right] \left( \frac{\Delta(\tau)}{\Delta(0)} \right) d\tau,$$

which for long times decays as $t^{-2K}$. Thus, an initially contacted state changes the power-law decay from $t^{-2K}$ to the slower $t^{-2K}$. The amplitude of the transient oscillations is also significantly different, due to the factor $(2K - 1)^{-1}$ in the above equation. For $K=0.75$, $j_{T_1}$ oscillates with an amplitude $\sim 10$ times larger than that of $j_{T_0}$, see Fig. 2. The magnitude of the oscillations was unexpected since for $j_{T_1}$ we only switch a bias while for $j_{T_0}$ also the contacts. This effect is not an artifact of the perturbative treatment: to support the validity of our results we checked that for $\eta_T = 1$ and zero bias the density matrix $\rho(t) = \langle \Psi_0, \Psi_{R,H_L}(t) \Psi_{L,H}(t) | \Psi_0 \rangle$ does not evolve in time to first order in $\lambda$ (this is obvious for the exact density matrix). The constant value $\rho(t) = \rho(0)$ is the result of a subtle cancellation of TD functions similar to $A_1(t)$ and $B_1(t)$.

**Correlated versus uncorrelated ground state.**—Next we consider the effects of correlations in the GS. We take $\eta_T = 1$ and compare the TD currents $j_{T_1}$ and $j_{T_0}$ resulting from Eq. (5) when $\eta_T = 1$ and $\eta_T = 0$, respectively. Note that $j_{T_1} = j_{T_0}$ (already calculated above). The current $j_{T_0} = \xi \text{Re}[A_0 + B_0]$ is the response to a sudden bias switching and interaction quench: at $t > 0$ the electrons start tunneling from $L$ to $R$ and at the same time forming interacting quasiparticles. The interaction quench has a dramatic impact on the transport properties, both in the transient and steady-state regimes. From Fig. 3 we clearly see that the relaxation behavior is different. The damping envelope of $j_{T_0}(t)$ is $\propto t^{-2K-1}$ as opposed to $t^{-2K}$ of $j_{T_1}(t)$.

In the long-time limit we find the intriguing result that $j_{T_0}(t \to \infty)$ is exactly given by Eq. (8) with exponent $\beta = K^2$, thus suggesting that the structure of the formula (8) is universal. Below we will prove that this is indeed the case and that $\beta$ is an elegant functional of the switching process. For now, we observe that ground-state correlations are not reproducible by quenching the interaction. The system remembers them forever and steady-state quantities are inevitably affected. This behavior is reminiscent of the thermalization breakdown enlightened by Cazalilla [10] and others [11,12]. Here, however, we are neither in equilibrium nor close to it (the bias is treated to all orders). The nonequilibrium exponents $\beta = 2K - 1$ and $\beta = K^2$ refer to current-carrying states as obtained from the full TD Schrödinger equation with different initial states.

**History dependence.**—We now address the question whether or not the physical steady-current $j_3(2K - 1)$ of Eq. (8) is reproducible by more sophisticated switching processes of the interaction like, e.g., an adiabatic switching. Preliminary insight can be gained by calculating $j(t)$ for a double quench: we first quench an interaction with $K_1 = (1 + K)/2$, let the system evolve, and then change suddenly $K_1 \to K_2 = K$. The current is calculated along the same line of reasoning of Eq. (4), although the formulas become considerably more cumbersome. In Fig. 4 we compare the TD currents for initially uncontacted leads resulting from an interaction $K$ (solid), a single quench $1 \to K$ (dashed), and the aforementioned double quench (dotted). We clearly see that in the latter case the steady current is larger than $j_3(K^2)$ (single quench) and gets closer to $j_3(2K - 1)$. Strikingly, the double-quench steady current is again given by $j_3(\beta)$ of Eq. (8) with $\beta = \frac{1}{2}(1 + K_i^2)[1 + (K_i^2)] - 1$. This value of $\beta$ depends only on the $K$ sequence and is independent of the quenching times. We have been able to extend the above solution to systems initially interacting with $K_0$ and then subject to...
an arbitrary sequence of quenches $K_0 \rightarrow K_1 \rightarrow \ldots \rightarrow K_N = K$. We found the remarkable result that the formula (8) is universal, with the sequence dependent $\beta$ given by

$$\beta[K_n] = \frac{K_0}{2^{N-1}} \prod_{n=0}^{N-1} \left[ 1 + \left( \frac{K_{n+1}}{K_n} \right)^2 \right] - 1. \quad (10)$$

This formula yields the correct values of $\beta$ for the single and double quench discussed above. Note that for a sequence of increasing interactions $K_{n+1} \leq K_n$, it holds $\beta \geq 2K - 1$ with the equality valid only for $K_0 = K_1 = \ldots = K_N = K$.

We now show that the special value $\beta = 2K - 1$ is also reproducible by an arbitrary (not necessarily adiabatic) continuous ($N \rightarrow \infty$) sequential quenching. In this limit the variable $x_n = n/N$ becomes a continuous variable and we can think of the $K_n$ as the values taken by a differentiable function $K(x)$ in $x = x_n$, with $K(0) = K_0$ and $K(1) = K$. Then, the quantity $\beta$ becomes a functional of $K(x)$ that we now work out explicitly. Approximating $K(x_{n+1}) = K(x_n + \frac{1}{N}) \approx K(x_n) + \frac{1}{N} K'(x_n)$ and taking the logarithm of Eq. (10) we can write

$$\log \left( \beta[K(x)] + 1 \right) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} \log \left( 1 + \frac{1}{N} K'(x_n) \right) = \int_0^1 dx \frac{K'(x)}{K(x)} = \log \frac{K(1)}{K(0)}, \quad (11)$$

from which it follows the history independent result

$$\beta[K(x)] = 2K - 1. \quad (12)$$

The above result can easily be generalized to discontinuous switching functions $K(x)$ for which, instead, the exponent $\beta$ is history dependent.

**Conclusions.**—In conclusion we studied the role of different preparative configurations in TD quantum transport between LLs. By using bosonization methods we showed that a sudden switching of the contacts alters significantly the transient behavior (changing the damping envelope from $\sim t^{-2K}$ to $\sim t^{1-2K}$) and magnifies the amplitude of the oscillations while preserving the same steady state. The effects of a sudden interaction quench is even more striking. Besides a different power-law decay ($\sim t^{1-2K}$ versus $\sim t^{-K-1}$ damping envelope) the steady current is also different; the $I$-$V$ characteristic $J_S \propto V^\beta$ changes from $\beta = 2K - 1$ to $\beta = K^2$. More generally we proved that for a sequence of interaction quenches the steady current is a universal function of the exponent $\beta$ which, in turn, is a functional of the switching process. It is only for smooth switchings that $\beta$ is history independent and equals the value $2K - 1$ of the initially interacting LL. The explicit $\beta$ functional derived in this Letter establishes the existence of intriguing memory effects that point to a complex entanglement between equilibrium and nonequilibrium correlations in strongly confined systems.