Yangian Symmetry and a New Regularization of Scattering Amplitudes in $\mathcal{N} = 4$ Super Yang-Mills

Jan Plefka

Humboldt-Universität zu Berlin

based on work with

James Drummond and Johannes Henn, JHEP 0905, arXiv:0902.2987
Fernando Alday, Johannes Henn and Theodor Schuster, JHEP 1001, arXiv:0908.0684

Tor Vergata, Rome, 22.01.2010

The setting

AdS/CFT correspondence: Fascinating link between conformal quantum field theories without gravity and string theory a theory with gravity (both classical and quantized)

Two major (recent) developments in the maximal susy $\text{AdS}_5/\text{CFT}_4$ system:

4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

1. Integrability in AdS/CFT:
   $\Rightarrow$ (close) to solution of the spectral problem: Scaling dims alias string spectrum
2. Scattering amplitudes in maximally susy Yang-Mills,
   $\Rightarrow$ relation to light-like Wilson loops
   $\Rightarrow$ emergence of dual superconformal symmetry

This talk: Can we connect the two?

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ, $\mu = 0, 1, 2, 3$, $\Delta = 1$
  - 6 real scalars: Φ_I, $I = 1, \ldots, 6$, $\Delta = 1$
  - 4×4 real fermions: $\Psi_{\alpha\, A}$, $\bar{\Psi}^{\dot{\alpha}}_{A}$,$\alpha, \dot{\alpha} = 1, 2$. $A = 1, 2, 3, 4$, $\Delta = 3/2$
  - Covariant derivative: $\mathcal{D}_\mu = \partial_\mu - i[A_\mu, *]$, $\Delta = 1$

- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\mathrm{YM}}}^2} \int d^4x \,\mathrm{Tr}\Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4}[\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot{\alpha}} \sigma^{\dot{\alpha}\beta}_\mu \mathcal{D}^\mu \Psi_{\beta\, A} - \tfrac{i}{2} \Psi_{\alpha\, A} \sigma^{AB}_I \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta\, B}] - \tfrac{i}{2} \bar{\Psi}_{\dot{\alpha}\, A} \sigma^{AB}_I \epsilon^{\dot{\alpha}\dot{\beta}} [\Phi^I, \bar{\Psi}_{\dot{\beta}\, B}]\Big]$$

- $\boxed{\beta_{g_{\mathrm{YM}}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\mathrm{YM}}}^2 N$

- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.

Most symmetric 4d gauge theory!

- Symmetry: $\mathfrak{so}(2,4) \otimes \mathfrak{so}(6) \subset \mathfrak{psu}(2,2|4)$

Poincaré: $p^{\alpha\dot{\alpha}} = p_\mu \, (\sigma^\mu)^{\dot{\alpha}\beta}, \quad m_{\alpha\beta}, \quad \bar{m}_{\dot{\alpha}\dot{\beta}}$

Conformal: $k_{\alpha\dot{\alpha}}, \quad d \qquad$ (c : central charge)

R-symmetry: r_{AB}

Poncaré Susy: $q^{\alpha\, A}, \bar{q}^{\dot{\alpha}}_A \qquad$ Conformal Susy: $s_{\alpha\, A}, \bar{s}^A_{\dot{\alpha}}$

- $4+4$ Supermatrix notation $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} m^\alpha{}_\beta - \frac{1}{2}\,\delta^\alpha_\beta\,(d + \frac{1}{2}c) & k^\alpha{}_{\dot{\beta}} & s^\alpha{}_B \\ p^{\dot{\alpha}}{}_\beta & \overline{m}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\,\delta^{\dot{\alpha}}_{\dot{\beta}}\,(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_B \\ q^A{}_\beta & \bar{s}^A{}_{\dot{\beta}} & -r^A{}_B - \frac{1}{4}\delta^A_B\, c \end{pmatrix}$$

- Algebra:

$$[J^{\bar{A}}{}_{\bar{B}}\,,\, J^{\bar{C}}{}_{\bar{D}}\} = \delta^{\bar{C}}_{\bar{B}}\, J^{\bar{A}}{}_{\bar{D}} - (-1)^{(|\bar{A}|+|\bar{B}|)(|\bar{C}|+|\bar{D}|)}\delta^{\bar{A}}_{\bar{D}}\, J^{\bar{C}}{}_{\bar{B}}$$

Gauge Theory Observables

- **Scaling dimensions:**
  Local operators $\mathcal{O}_n(x) = \mathrm{Tr}[\mathcal{W}_1\,\mathcal{W}_2\ldots\mathcal{W}_n]$ with $\mathcal{W}_i \in \{\mathcal{D}^k\Phi, \mathcal{D}^k\Psi, \mathcal{D}^k F\}$

$$\langle \mathcal{O}_a(x_1)\,\mathcal{O}_b(x_2)\rangle = \frac{\delta_{ab}}{(x_1-x_2)^{2\,\Delta_a(\lambda)}} \qquad \Delta_a(\lambda) = \sum_{l=0}^{\infty} \lambda^l\,\Delta_{a,l}$$

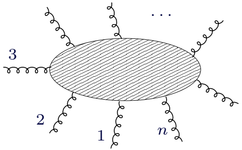
- **Wilson loops:**

$$\mathcal{W}_C = \left\langle \mathrm{Tr} P \exp i \oint_C ds\,(\dot{x}^\mu A_\mu + i|\dot{x}|\,\theta^I\,\Phi_I) \right\rangle$$

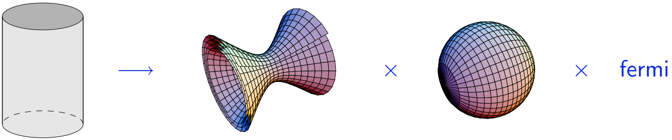
- **Scattering amplitudes:**

$$\mathcal{A}_n(\{p_i, h_i, a_i\};\lambda) = \left\{ \begin{matrix} \text{UV-finite} \\ \text{IR-divergent} \end{matrix} \right\}$$

helicities: $\quad h_i \in \{0, \pm\frac{1}{2}, \pm 1\}$



Superstring in $AdS_5 \times S^5$



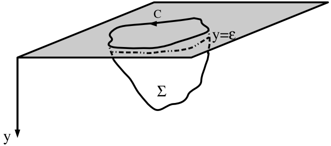
$$I = \sqrt{\lambda} \int d\tau\, d\sigma \left[G^{(\mathrm{AdS}_5)}_{mn}\, \partial_a X^m \partial^a X^n + G^{(\mathrm{S}_5)}_{mn}\, \partial_a Y^m \partial^a Y^n + \text{fermions} \right]$$

- $ds^2_{AdS} = R^2 \dfrac{dx^2_{3+1} + dz^2}{z^2}$ has boundary at $z = 0$
- $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, classical limit: $\sqrt{\lambda} \to \infty$, quantum fluctuations: $\mathcal{O}(1/\sqrt{\lambda})$
- $AdS_5 \times S^5$ is max susy background (like $\mathbb{R}^{1,9}$ and plane wave)
- **Quantization unsolved!**
- String coupling constant $g_s = \frac{\lambda}{4\pi N} \to 0$ in 't Hooft limit
- Isometries: $\mathfrak{so}(2,4) \times \mathfrak{so}(6) \subset \mathfrak{psu}(2,2|4)$
- Include fermions: Formulate as $\frac{PSU(2,2|4)}{SO(1,4)\times SO(5)}$ supercoset model [Metsaev,Tseytlin]

Gauge Theory - String Theory Dictionary of Observables

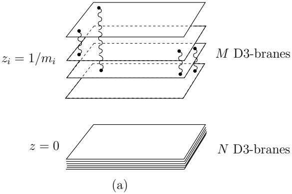
$\Delta_a(\lambda)$ spectrum of scaling dimensions $\Leftrightarrow$ $E(\lambda)$ string excitation spectrum — solved (?)

Wilson loop $\mathcal{W}_C$ $\Leftrightarrow$



minimal surface

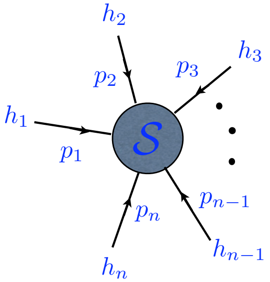
$\mathcal{A}_n(\{p_i, h_i, a_i\}; \lambda)$ $(\Leftrightarrow)$



open string amps

Scattering amplitudes in $\mathcal{N} = 4$ SYM I

- Consider n-particle scattering amplitude



Planar amplitudes most conveniently expressed in color ordered formalism:

$$A_n(\{p_i, h_i, a_i\}) = (2\pi)^4\, \delta^{(4)}(\sum_{i=1}^{n} p_i) \sum_{\sigma \in S_n/Z_n} g^{n-2}\, \mathrm{tr}[t^{a_1} \ldots t^{a_n}]$$
$$\times\, \mathcal{A}_n(\{p_{\sigma_1}, h_{\sigma_1}\}, \ldots, \{p_{\sigma_1}, h_{\sigma_1}\}; \lambda = g^2\, N)$$

$\mathcal{A}_n$: Color ordered amplitude. Color structure is stripped off.

Helicity of ith particle: $h_i = 0$ scalar, $h_i = \pm 1$ gluon, $h_i = \pm\frac{1}{2}$ gluino

Spinor helicity formalism

- Express momentum and polarizations via commuting spinors λ^α, $\tilde\lambda^{\dot\alpha}$:

$$p^{\alpha\dot\alpha} = (\sigma^\mu)^{\alpha\dot\alpha}\, p_\mu = \lambda^\alpha \tilde\lambda^{\dot\alpha} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot\alpha} = 0$$

- Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot\alpha} = \frac{\lambda^\alpha \tilde\mu^{\dot\alpha}}{[\tilde\lambda\, \tilde\mu]} \qquad [\tilde\lambda\, \tilde\mu] := \epsilon_{\dot\alpha\dot\beta}\, \tilde\lambda^{\dot\alpha} \tilde\mu^{\dot\beta}$$

$$h = -1 \qquad \tilde\varepsilon^{\alpha\dot\alpha} = \frac{\mu^{\alpha}\tilde\lambda^{\dot\alpha}}{\langle\lambda\, \mu\rangle} \qquad \langle\lambda\, \mu\rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

  $\mu, \bar\mu$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle\lambda_1, \lambda_2\rangle\, [\tilde\lambda_2, \tilde\lambda_1] = \langle 1, 2\rangle\, [2, 1]$

Scattering amplitudes in $\mathcal{N} = 4$ SYM II

- Gluon amplitudes: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ by SUSY Ward identities
- **Maximally helicity violating (MHV) amplitudes**

$$\mathcal{A}_n(1^-, 2^+, \ldots, (j-1)^+, j^-, (j+1)^+, \ldots n^+) = \mathcal{A}_{n;0}^{\text{MHV}} + \lambda \cdot \mathcal{A}_{n;1}^{\text{MHV}} + \lambda^2 \cdot \mathcal{A}_{n;2}^{\text{MHV}} + \ldots = \mathcal{A}_{n;0}^{\text{MHV}} \cdot \mathcal{M}_n^{\text{MHV}}(\{p_i \cdot p_j\}; \lambda)$$

Parke-Taylor formula: $$\boxed{\mathcal{A}_{n;0}^{\text{MHV}} = i \frac{\langle 1, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}}$$ [Parke, Taylor]

- BDS conjecture [Anastasiou, Bern, Dixon, Kosower; Bern, Dixon, Smirnov]

$$\boxed{\log \mathcal{M}_n^{\text{MHV}} = \Gamma_{\text{cusp}}(\lambda) \cdot \mathcal{M}_{n,\text{1-loop,finite}}^{\text{MHV}} + \text{“}\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\text{”}}$$

True for $n = 4, 5$ known to receive corrections for $n \geq 6$

[Drummond, Henn, Korchemsky, Sokatchev; Bern, Dixon, Kosower, Roiban, Spradlin, Vergu, Volovich]

$\Rightarrow$ We know **all** 4 and 5 point amplitudes to all loop order!

- N^{k}MHV amplitudes have rather complicated structure! $\Rightarrow$ Better formulation?

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  by SUSY Ward identities
- **Maximally helicity violating (MHV) amplitudes**

$$\mathcal{A}_n(1^-, 2^+, \ldots, (j-1)^+, j^-, (j+1)^+, \ldots n^+) =$$
$$\mathcal{A}^{\text{MHV}}_{n;0} + \lambda \cdot \mathcal{A}^{\text{MHV}}_{n;1} + \lambda^2 \cdot \mathcal{A}^{\text{MHV}}_{n;2} + \ldots = \mathcal{A}^{\text{MHV}}_{n;0} \cdot \mathcal{M}^{\text{MHV}}_n(\{p_i \cdot p_j\}; \lambda)$$

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On-shell superspace

- Introduce Grassmann variables η_i^A $\quad A = 1,2,3,4 \quad i = 1,\dots,n$ [Nair]
- Superwavefunction:

$$\Phi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Express amplitudes compactly in on-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$
- MHV-superamplitude: Packaged gluon$^\pm$-gluino$^{\pm 1/2}$-scalar amplitude

$$\mathbb{A}_{n;0}^{\text{MHV}}(\lambda_1, \tilde{\lambda}_1, \eta_1; \dots; \lambda_n, \tilde{\lambda}_n, \eta_n) = i(2\pi)^4 \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle}$$

Conservation of 'fermionic' momentum: $\quad \delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

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Superamplitudes and $\mathfrak{su}(2,2|4)$ invariance

- General form of **superamplitudes**:

$$\mathbb{A}_n = i(2\pi)^4 \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^{n} \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^{n} \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{ obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^{n} \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^{n} \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{ less obvious sym}$$

- Invariance: $\{\, p, k, \bar{m}, m, d, r, q, \bar{q}, s, \bar{s}, c_i \,\}\, \mathbb{A}_n^{\text{tree}}(\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A) = 0$
- N.B.: **Local** invariance $\quad h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

  Helicity operator: $\quad h_i = -\frac{1}{2}\, \lambda_i^\alpha\, \partial_{i\,\alpha} + \frac{1}{2}\, \tilde{\lambda}_i^{\dot{\alpha}}\, \partial_{i\,\dot{\alpha}} + \frac{1}{2}\, \eta_i^A\, \partial_{i\,A} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- The $\mathfrak{su}(2,2|4)$ generators acting in on-shell superspace $(\lambda_i^\alpha, \tilde\lambda_i^{\dot\alpha}, \eta_i^A)$:

$$p^{\dot\alpha\alpha} = \sum_i \tilde\lambda_i^{\dot\alpha}\lambda_i^\alpha\,, \qquad k_{\alpha\dot\alpha} = \sum_i \partial_{i\alpha}\partial_{i\dot\alpha}\,,$$

$$\overline{m}_{\dot\alpha\dot\beta} = \sum_i \tilde\lambda_{i(\dot\alpha}\partial_{i\dot\beta)}, \qquad m_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)}\,,$$

$$d = \sum_i [\tfrac{1}{2}\lambda_i^\alpha\partial_{i\alpha} + \tfrac{1}{2}\tilde\lambda_i^{\dot\alpha}\partial_{i\dot\alpha} + 1], \qquad r^A{}_B = \sum_i [-\eta_i^A\partial_{iB} + \tfrac{1}{4}\delta_B^A\eta_i^C\partial_{iC}]\,,$$

$$q^{\alpha A} = \sum_i \lambda_i^\alpha\eta_i^A\,, \qquad \bar q_A^{\dot\alpha} = \sum_i \tilde\lambda_i^{\dot\alpha}\partial_{iA}\,,$$

$$s_{\alpha A} = \sum_i \partial_{i\alpha}\partial_{iA}, \qquad \bar s_{\dot\alpha}^A = \sum_i \eta_i^A\partial_{i\dot\alpha}\,,$$

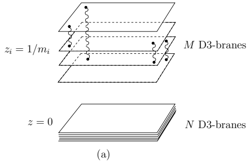
$$c = \sum_i [1 + \tfrac{1}{2}\lambda_i^\alpha\partial_{i\alpha} - \tfrac{1}{2}\tilde\lambda_i^{\dot\alpha}\partial_{i\dot\alpha} - \tfrac{1}{2}\eta_i^A\partial_{iA}]\,.$$

- Invariance: $\{\,p, k, \bar m, m, d, r, q, \bar q, s, \bar s, c_i\,\}\,\mathbb{A}_n^{\text{tree}}(\lambda_i^\alpha, \tilde\lambda_i^{\dot\alpha}, \eta_i^A) = 0$
- N.B: Subtleties for colinear momenta due to holomorphic anomalies

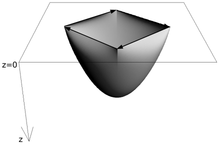
[Bargheer, Beisert, Galleas, Loebbert,McLoughlin][Korchemsky, Sokatchev] [Skinner,Mason][Arkani-Hamed, Cachazo, Kaplan]

MHV Scattering amplitudes in AdS/CFT

- Dual string description of scattering amplitudes [Alday,Maldacena '07]



T-dual $\Leftrightarrow$



Open string amplitude on IR-branes $\overset{\text{T-dual}}{\Leftrightarrow}$ Wilson loop with light-like segments

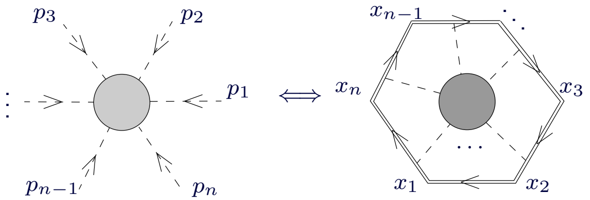
- Cusp points determined by gluon momenta via key relation

$$\boxed{p_i^\mu = x_{i+1}^\mu - x_i^\mu}$$

- Yields strong coupling prediction for four-gluon MHV amplitude via classical string theory!
- Indeed BDS conjecture for $n = 4$ gluons tested:

$$\lim_{\lambda\to\infty} \log \mathcal{M}_4^{\text{MHV}} = \underbrace{\sqrt{\lambda}/2\pi}_{\Gamma_{\text{cusp}}(\lambda\to\infty)} \cdot \mathcal{M}_{4,\text{1-loop}}^{\text{MHV}} + \text{``}\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\text{''}$$

Scattering amplitude $\Leftrightarrow$ Wilson loop duality at perturbative level



$$x^\mu_{i+1} - x^\mu_i = p^\mu_i$$

[Drummond,Henn,Korchemsky, Sokatchev]

Planar relation:

$$\ln \mathcal{M}_n^{\mathsf{MHV}} = \ln \mathcal{W}_n + \mathsf{const} + \mathcal{O}(\epsilon)$$

$$\mathcal{W}_n = \frac{1}{N} \left\langle \mathrm{Tr} P \exp[ig \oint_{C_n} dx^\mu A_\mu] \right\rangle$$

Checked up to two loops and $n \leq 6$ points.

[Drummond,Henn,Korchemsky,Sokatchev;Brandhuber,Heslop,Travaglini; Bern,Dixon,Kosower,Roiban,Spradlin,Vergu,Volovich]

String interpretation: Combination of bosonic and 'fermionic' T-duality transformation for $AdS_5 \times S^5$ superstring.

[Beisert,Ricci,Tseytlin,Wolf;Berkovits,Maldacena]

Conformal invariance in dual space $\Rightarrow$ Dual conformal symmetry of scattering amps!

Dual Superconformal symmetry

- Introduce dual on-shell superspace [Drummond, Henn, Korchemsky, Sokatchev]

$$(x_i - x_{i+1})^{\alpha\dot\alpha} = \lambda_i^\alpha\,\tilde\lambda_i^{\dot\alpha} \qquad (\theta_i - \theta_{i+1})^{\alpha\,A} = \lambda_i^\alpha\,\eta_i^A$$

  Then $x_i^{\alpha\dot\alpha}$ and $\theta_i^{\alpha\,A}$ have standard transformation law under (dual) conformal transformations

- Representation of dual superconf. algebra, $\{\,P, M, \bar M, K, D \oplus R \oplus Q, \bar Q, S, \bar S\,\}$, acting in dual on-shell superspace $(x^{\alpha\dot\alpha}, \theta^{\alpha A})$:

$$P_{\alpha\dot\alpha} = \sum_i \partial_{i\alpha\dot\alpha}\,, \qquad Q_{\alpha A} = \sum_i \partial_{i\alpha A}$$

$$K^{\alpha\dot\alpha} = \sum_i x_i^{\alpha\dot\beta}\,x_i^{\dot\alpha\beta}\,\frac{\partial}{\partial x_i^{\beta\dot\beta}} + x_i^{\dot\alpha\beta}\,\theta_i^{\alpha\,B}\,\frac{\partial}{\partial\theta_i^{\beta\,B}}$$

$$S_\alpha^A = \sum_i -\theta_{i\alpha}^B\theta_i^{\beta A}\partial_{i\beta B} + x_{i\alpha}{}^{\dot\beta}\theta_i^{\beta A}\partial_{\beta\dot\beta}$$

$$\vdots$$

The natural question

Q: What algebraic structure emerges when one commutes conformal with dual conformal generators? [Drummond,Henn,Plefka]

First step: Express dual superspace coordinates (x_i, θ_i) in terms of on-shell superspace coordinates $(\lambda_i, \tilde{\lambda}_i, \eta_i)$!

1. Open chain by droping $x_{n+1} = x_1$ and $\theta_{n+1} = \theta_1$ conditions, implemented via δ-fcts: $\delta^{(4)}(p)\,\delta^{(8)}(q) = \delta^{(4)}(x_1 - x_{n+1})\,\delta^{(8)}(\theta_1 - \theta_{n+1})$
2. Express dual variables via "non-local' relations:

$$x_i^{\alpha\dot{\alpha}} = x_1^{\alpha\dot{\alpha}} + \sum_{j<i} \lambda_j^\alpha\,\tilde{\lambda}_j^{\dot{\alpha}} \qquad \theta_i^{\alpha A} = \theta_1^{\alpha A} + \sum_{j<i} \lambda_j^\alpha\,\eta_j^A$$

   Now set $x_1 = \theta_1 = 0$ by dual translation P and Poincare Susy Q.

3. Eliminate all x_i and θ_i derivatives in dual superconformal generators.

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   Now set $x_1 = \theta_1 = 0$ by dual translation P and Poincare Susy Q.

3. Eliminate all x_i and θ_i derivatives in dual superconformal generators.

Dual $\mathfrak{psu}(2,2|4)$ generators

- Dual superconformal generators acting in standard on-shell superspace $(\lambda, \tilde{\lambda}, \eta)$:

$$P_{\alpha\dot{\alpha}} = 0\,, \qquad Q_{\alpha A} = 0\,, \qquad \overline{Q}_{\dot{\alpha}}^{A} = \sum_i \eta_i^A \partial_{i\dot{\alpha}} = \bar{s}_{\dot{\alpha}}^{A}$$

$$M_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} = \bar{m}_{\dot{\alpha}\dot{\beta}}\,, \qquad \overline{M}_{\dot{\alpha}\dot{\beta}} = \sum_i \tilde{\lambda}_{i(\dot{\alpha}}\partial_{i\dot{\beta})} = m_{\alpha\beta}\,,$$

$$R^A{}_B = \sum_i \eta_i^A\,\partial_{iB} - \tfrac{1}{4}\delta_B^A \eta_i^C \partial_{iC} = -r^A{}_B\,,$$

$$D = \sum_i -\tfrac{1}{2}\lambda_i^\alpha \partial_{i\alpha} - \tfrac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}}\partial_{i\dot{\alpha}} = -d\,,$$

$$C = \sum_i -\tfrac{1}{2}\lambda_i^\alpha \partial_{i\alpha} + \tfrac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}}\partial_{i\dot{\alpha}} + \tfrac{1}{2}\eta_i^A \partial_{iA} = 1 - c\,,$$

$$S_\alpha^A = \sum_i \lambda_{i\alpha}\theta_i^{\gamma A}\partial_{i\gamma} + x_{i+1\,\alpha}{}^{\dot{\beta}}\eta_i^A \partial_{i\dot{\beta}} - \theta_{i+1\,\alpha}^{B}\eta_i^A\partial_{iB} \qquad \leftarrow \text{new}$$

$$\overline{S}_{\dot{\alpha}A} = \sum_i \tilde{\lambda}_{i\dot{\alpha}}\partial_{iA} = \bar{q}_{\dot{\alpha}A}\,,$$

$$K_{\alpha\dot{\alpha}} = \sum_i x_{i\dot{\alpha}}{}^{\beta}\lambda_{i\alpha}\partial_{i\beta} + x_{i+1\,\alpha}{}^{\dot{\beta}}\tilde{\lambda}_{i\dot{\alpha}}\partial_{i\dot{\beta}} + \tilde{\lambda}_{i\dot{\alpha}}\theta_{i+1\,\alpha}^{B}\partial_{iB} \qquad \leftarrow \text{new}$$

Nonlocal structure of dual K and S

- We are left with the dual generators K and S, all others trivially related to standard superconformal generators.

$$K^{\alpha\dot{\alpha}} = \sum_{i=1}^{n} x_i^{\dot{\alpha}\beta}\,\lambda_i^{\alpha}\,\frac{\partial}{\partial\lambda_i^{\beta}} + x_{i+1}^{\alpha\dot{\beta}}\,\tilde{\lambda}_i^{\dot{\alpha}}\,\frac{\partial}{\partial\tilde{\lambda}_i^{\dot{\beta}}} + \tilde{\lambda}_i^{\dot{\alpha}}\,\theta_{i+1}^{\alpha B}\,\frac{\partial}{\partial\eta_i^{B}} + x_i^{\alpha\dot{\alpha}}$$

$$x_i^{\alpha\dot{\alpha}} = \sum_{j=1}^{i-1}\lambda_j^{\alpha}\,\tilde{\lambda}_j^{\dot{\alpha}} \qquad \theta_{i+1}^{\alpha A} = \sum_{j=1}^{i}\lambda_j^{\alpha}\,\eta_j^{A}$$

Nonlocal structure!

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Can show that dual superconformal generators K and S may be lifted to level 1 generators of a Yangian algebra $Y[\mathfrak{psu}(2,2|4)]$:

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators

$$\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

$$[J_a^{(1)}, [J_b^{(1)}, J_c^{(0)}\}\} + (-1)^{|a|(|b|+|c|)}[J_b^{(1)}, [J_c^{(1)}, J_a^{(0)}\}\} + (-1)^{|c|(|a|+|b|)}[J_c^{(1)}, [J_a^{(1)}, J_b^{(0)}\}\}$$
$$= h(-1)^{|r||m|+|t||n|}\{J_l^{(0)}, J_m^{(0)}, J_n^{(0)}] f_{ar}{}^l f_{bs}{}^m f_{ct}{}^n f^{rst}.$$

- An infinite dimensional Hopf algebra

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Bosonic invariance $\boxed{p^{(1)}_{\alpha\dot\alpha}\,\mathbb{A}_n = 0}$ with

$$p^{(1)}_{\alpha\dot\alpha} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j}(m_{i,\alpha}{}^{\gamma}\delta^{\dot\gamma}_{\dot\alpha} + \bar m_{i,\dot\alpha}{}^{\dot\gamma}\delta^{\gamma}_{\alpha} - d_i\,\delta^{\gamma}_{\alpha}\delta^{\dot\gamma}_{\dot\alpha})\,p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\,q^{C}_{j,\alpha} - (i\leftrightarrow j)$$

- In supermatrix notation: $\bar A = (\alpha,\dot\alpha|A)$

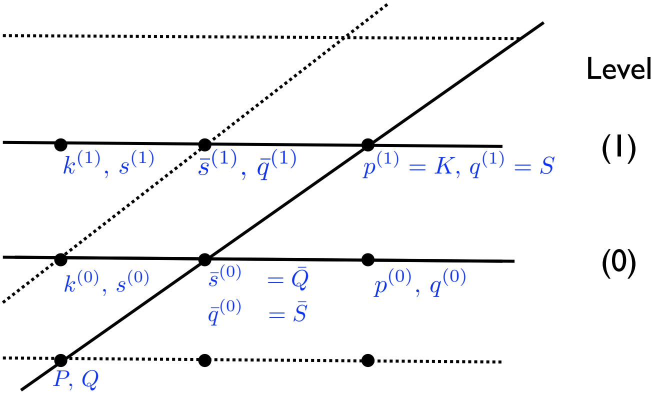
$$J^{\bar A}{}_{\bar B} = \begin{pmatrix} m^{\alpha}{}_{\beta} - \frac{1}{2}\,\delta^{\alpha}_{\beta}\,(d+\frac{1}{2}c) & k^{\alpha}{}_{\dot\beta} & s^{\alpha}{}_{B} \\ p^{\dot\alpha}{}_{\beta} & \overline{m}^{\dot\alpha}{}_{\dot\beta} + \frac{1}{2}\,\delta^{\dot\alpha}_{\dot\beta}\,(d-\frac{1}{2}c) & \bar q^{\dot\alpha}{}_{B} \\ q^{A}{}_{\beta} & \bar s^{A}{}_{\dot\beta} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar A}{}_{\bar B} := -\sum_{i>j}(-1)^{|\bar C|}(J_i^{\bar A}{}_{\bar C}\,J_j^{\bar C}{}_{\bar B} - J_j^{\bar A}{}_{\bar C}\,J_i^{\bar C}{}_{\bar B})}$$

- Implies an infinite-dimensional symmetry algebra for $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

Summary of Yangian Structure

- Combination of standard and dual superconformal symmetry lifts to Yangian $Y[\mathfrak{psu}(2,2|4)]$ [Picture: Beisert]



- Tree level superamplitudes invariant: $\boxed{\mathcal{J} \circ \mathbb{A}_n^{\text{tree}} = 0}$ for $\mathcal{J} \in Y[\mathfrak{psu}(2,2|4)]$.

Higher loops

- Beyond tree-level: Conformal and dual conformal symmetry is broken by IR divergencies $\Rightarrow \{\not{s}, \not{\bar{s}}, \not{k}, \not{K}, \not{S}, \not{\bar{Q}}\}$
- Need for regularization: Standard method Dim reduction $10 \to 4 - \epsilon$
- Specialize to MHV for simpicity: $\boxed{\mathcal{A}_n^{\text{MHV}} = \mathcal{A}_{n,0}^{\text{MHV}}\, \mathcal{M}_n^{\text{MHV}}(p_i \cdot p_j; \lambda)}$
- All loop planar amplitudes can be split into IR divergent and finite parts:

$$\ln \mathcal{M}_n^{\text{MHV}} = D_n + F_n + \mathcal{O}(\epsilon)$$

IR divergencies exponentiate in any gauge theory $(a = \lambda/8\pi^2)$ [Mueller,Collins,Sterman,. . .]

$$D_n = -\frac{1}{2}\sum_{l=1}^{\infty} a^l \left(\frac{\Gamma_{\text{cusp}}^{(l)}}{(l\epsilon)^2} + \frac{G^{(l)}}{l\epsilon} \right) \sum_{i=1}^{n} (2p_i \cdot p_j)^{l\epsilon}$$

$$\Gamma_{\text{cusp}}(a) = \sum_l a^l \Gamma_{\text{cusp}}^{(l)}\,, \quad \text{cusp anomalous dimension}$$

$$G(a) = \sum_l a^l G^{(l)}\,, \quad \text{colinear anomalous dimension}$$

Dual Conformal Anaomaly

- Breaking of K_μ is under control: Ward identity from dual Wilson loop (UV anomaly due to cusps) [Drummond,Henn,Korchemsky,Sokatchev]

$$K_\mu F_n = \sum_{i=1}^{n} \left[2x_{i\mu} x_i^\nu \frac{\partial}{\partial x_i^\nu} - x_i^2 \frac{\partial}{\partial x_i^\mu} \right] F_n = \frac{1}{2} \Gamma_{\text{cusp}}(a) \sum_{i=1}^{n} \left[x_{i,i+1}^\mu \ \ln \frac{x_{i,i+2}^2}{x_{i-1,i+1}^2} \right] F_n$$

- Checked at one-loop for all MHV and non-MHV amplitudes [Brandhuber,Heslop,Travaglini]
- 'Anomaly' fixes the MHV 4 & 5 gluon amplitudes completely. Nontrivial structure starts with $n = 6$.
- **Q:** Can the other broken generators, in particular the standard conformal generators $\{s, \bar{s}, k, d\}$ be similarly repaired at loop level?

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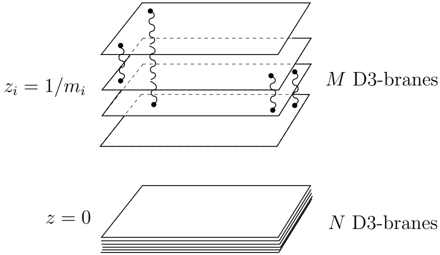
An alternative regularization [Alday, Henn, Plefka, Schuster]

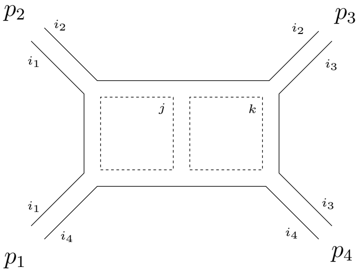
- Take string picture serious:



(a)

- Field Theory: Higgsing $U(N+M) \to U(N) \times U(1)^M$. One brane for every scattered particle, $N \gg M$.

Renders amplitudes IR finite. Have light $(m_i - m_j)$ and heavy m_i fields



Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$\begin{aligned} J_{-1,4} &= r\partial_r + x^\mu \partial_\mu = \hat{D} \\ J_{4,\mu} - J_{-1,\mu} &= \partial_\mu = \hat{P}_\mu \\ J_{4,\mu} + J_{-1,\mu} &= 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu \end{aligned}$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Higgsing $\mathcal{N} = 4$ Super Yang-Mills

$$\hat{S}^{U(N+M)}_{\mathcal{N}=4} = \int d^4x\, \mathrm{Tr}\Big(-\frac{1}{4}\,\hat{F}^2_{\mu\nu} - \frac{1}{2}(D_\mu \hat{\Phi}_I)^2 + \frac{g^2}{4}\,[\hat{\Phi}_I, \hat{\Phi}_J]^2 + \text{ferms}\Big)\,,$$

Decompose into $N + M$ blocks

$$\hat{A}_\mu = \begin{pmatrix} (A_\mu)_{ab} & (A_\mu)_{aj} \\ (A_\mu)_{ia} & (A_\mu)_{ij} \end{pmatrix}\,, \qquad \hat{\Phi}_I = \begin{pmatrix} (\Phi_I)_{ab} & (\Phi_I)_{aj} \\ (\Phi_I)_{ia} & \delta_{I9}\,\frac{m_i}{g}\,\delta_{ij} + (\Phi_I)_{ij} \end{pmatrix}$$

$$a, b = 1, \ldots, N\,, i, j = N+1, \ldots, N+M\,,$$

Add R_ξ gauge fixing and appropriate ghost terms. Qudratic terms $(A_M := (A_\mu, \Phi_I))$

$$\hat{S}_{\mathcal{N}=4}\Big|_{\text{quad}} = \int d^4x \Big\{ -\tfrac{1}{2}\mathrm{Tr}(\partial_\mu A_M)^2 - \tfrac{1}{2}(m_i - m_j)^2\,(A_M)_{ij}\,(A^M)_{ji}$$

$$- m_i^2\,(A_M)_{ia}\,(A^M)_{ai} + \text{ferms} \Big\}$$

Plus novel bosonic 3-point interactions

$$\hat{S}_{\mathcal{N}=4}\Big|_{\mathcal{O}(m_i)} = \int d^4x \Big\{ m_i\,([\Phi_9, A^\mu]\,A_\mu\,)_{ii} - m_i(A_\mu\,[\Phi_9, A^\mu]\,)_{ii}$$

$$+ m_i\,([\Phi_9, \Phi_{I'}]\,\Phi_{I'}\,)_{ii} - m_i\,(\Phi_{I'}\,[\Phi_9, \Phi_{I'}]\,)_{ii} \Big\}$$

Higgsing $\mathcal{N}=4$ Super Yang-Mills

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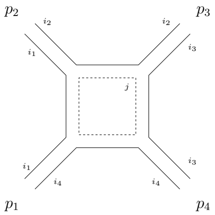
One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4) \rangle = i g^2_{\rm YM} \Big(1 + \lambda\, I^{(1)}(s,t,m_i) + O(a^2) \Big)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables ($p_i = x_i - x_{i+1}$)



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1-m_3)^2)(x_{24}^2 + (m_2-m_4)^2)}{(x_{15}^2+m_1^2)(x_{25}^2+m_2^2)(x_{35}^2+m_3^2)(x_{45}^2+m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\qquad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

One loop test of extended dual conformal symmetry 2

- Extended dual conformal invariance

$$\hat{K}_\mu \, I^{(1)}(s,t,m_i) := \sum_{i=1}^{4} \left[2x_{i\mu} \left(x_i^\nu \frac{\partial}{\partial x_i^\nu} + m_i \frac{\partial}{\partial m_i} \right) - (x_i^2 + m_i^2) \frac{\partial}{\partial x_i^\mu} \right] I^{(1)}(s,t,m_i) = 0$$

The only existing 4 particle invariants are: $\frac{m_1 m_3}{\hat{x}_{13}^2}$ and $\frac{m_2 m_4}{\hat{x}_{24}^2}$: One computes

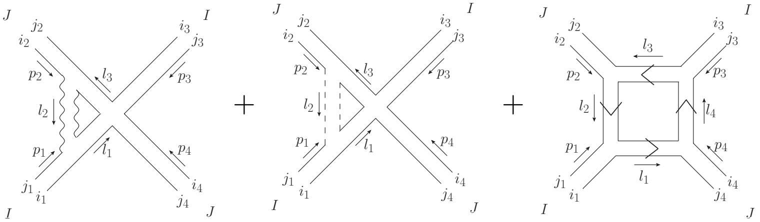
$$I^{(1)}(x_{13}^2, x_{24}^2, m_i) = f\left(\frac{m_1 m_3}{\hat{x}_{13}^2}, \frac{m_2 m_4}{\hat{x}_{24}^2} \right) = 2 \ln(\frac{m_1 m_3}{\hat{x}_{13}^2}) \ln(\frac{m_2 m_4}{\hat{x}_{24}^2}) - \pi^2 + O(m^2)$$

- Triangle and bubble graphs are forbidden by extended conformal symmetry!

One loop test of extended dual conformal symmetry 3

- Indeed an explicit one-loop calculation shows the cancelation of triangles:

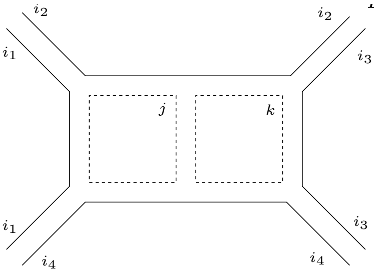
$$\langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4)\rangle_{\text{1-loop}} =$$



$$= -16 N g^4\, (\hat{p}_1 \cdot \hat{p}_2)(\hat{p}_3 \cdot \hat{p}_4) \int \frac{d^4 l}{(2\pi)^4}\, \frac{1}{\hat{l}_1^2\, \hat{l}_2^2\, \hat{l}_3^2\, \hat{l}_4^2} + \text{bubbles}$$

Extended dual conformal invariance at higher loops

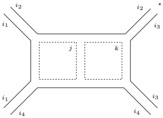
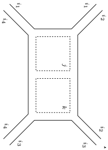
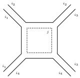
- At 2 loops: Only one integral is allowed by extended dual conformal symmetry:



  Should similarly restrict possible integrals at higher loops.
- Computed this graph in $m_i \to 0$ limit using Mellin-Barnes techniques.
- No $\frac{1}{\epsilon} \times \epsilon = 1$ 'interference' as in dimred: Here $\log(m^2) \times m^2 \to 0$.

Extracting the cusp anomalous dimension

- We have

$M_4\big|_{\text{2-loops}} =$  $+$  $= \exp\Big[\Gamma_{\text{cusp}}(\lambda)$  $\Big]\Big|_{\text{2-loops}}$

where one splits M_4 into $\ln m^2$ dependent and independent pieces:

$$\ln M_4 = D_4 + F_4 + \mathcal{O}(m^2)$$

- Defining $\boxed{\left(\frac{\partial}{\partial \ln(m^2)}\right)^2 \ln M_4 =: -\Gamma_{\text{cusp}}(a)}$ we find $\Gamma_{\text{cusp}}(a) = 2a - 2\zeta_2\, a^2 + \ldots$

where $a = \lambda/8\pi^2$ in agreement with dim reg.

- Furthermore for finite piece one has

$$F_4 = \frac{1}{2}\Gamma_{\text{cusp}}(a)\left[\frac{1}{2}\ln^2(s/t) + \frac{1}{2}\right] + C(a)$$

with $C(a) = a^2\,\pi^4/120 + \mathcal{O}(a^3)$.

Summary and Outlook

- Tree level amplitudes are invariant under standard superconformal and dual superconformal symmetry.
- Closure of both algebras yields infinite dimensional Yangian symmetry.

  $\Rightarrow$ Hint for integrability in scattering amplitudes!

- Is form of tree amplitudes fixed by Yangian symmetry?

  $\Rightarrow$ Needs to include colinear limits $\equiv$ length changing effects

  [Bargheer,Beisert,Galleas,Loebbert,McLoughlin]

- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Can be repaired by regularization via Higgsing with an exact extended dual conformal symmetry.
- Regularization via Higgsing restrict allowed integrals at higher loops. Is of pratical help for higher loop calculations, e.g. in Regge limit.
- Can breaking of standard conformal invariance at loop level be controlled?

  [Sever,Vieira]

- **Does integrability determine the all loop planar scattering amplitudes?**

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